**PGDM (IB) /PGDM (RM), 2014-16**

**Management Science / Operations Research**

**IB-304 / RM-306**

**Trimester – III, End-Term Examination: April 2015**

Time allowed: 2 hrs 30 min Max Marks: 50

Roll No: _____

**Instruction:** Students are required to write Roll No on every page of the question paper, writing anything except the Roll No will be treated as **Unfair Means.** In case of rough work please use answer sheet.

**Section – A**

**Attempt any 3 out of 5 questions from this section. Each question carries 5 marks.**

**Question 1** What is a shadow price? Describe how a firm would use the shadow price associated with a given constraint.

**Question 2** Consider Table summarizing the details of a project involving 14 activities.

| Activity | Immediate predecessor(s) | Duration (months) |
|---|---|---|
| A | -- | 2 |
| B | -- | 6 |
| C | -- | 4 |
| D | B | 3 |
| E | A | 6 |
| F | A | 8 |
| G | B | 3 |
| H | C, D | 7 |
| I | C, D | 2 |
| J | E | 5 |
| K | F, G, H | 4 |
| L | F, G, H | 3 |
| M | I | 13 |
| N | J, K | 7 |

Construct the CPM network.

**Question 3** Consider the assignment problem as shown in Table. In this problem, 5 different jobs are to be assigned to 5 different operators such that the total processing time is minimized. The matrix entries represent processing times in hours.

| | | Operator | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| Job | 1 | 10 | 12 | 15 | 12 | 8 |
| | 2 | 7 | 16 | 14 | 14 | 11 |
| | 3 | 13 | 14 | 7 | 9 | 9 |
| | 4 | 12 | 10 | 11 | 13 | 10 |
| | 5 | 8 | 13 | 15 | 11 | 15 |

**Question 4** Harley's Sand and Gravel Pit has contracted to provide topsoil for three residential housing developments. Topsoil can be supplied from three different "farms" as follows:

| Weekly Capacity | Farm (cubic yards) |
|---|---|
| A | 100 |
| B | 200 |
| C | 200 |

Demand for the topsoil generated by the construction projects is

| Weekly Demand | Project (cubic yards) |
|---|---|
| 1 | 50 |
| 2 | 150 |
| 3 | 300 |

The manager of the sand and gravel pit has estimated the cost per cubic yard to ship over each of the possible routes:

| | Cost per Cubic Yard to | | |
|---|---|---|---|
| **From** | **Project #1** | **Project #2** | **Project #3** |
| Farm A | $4 | $2 | $8 |
| Farm B | $5 | $1 | $9 |
| Farm C | $7 | $6 | $3 |

This constitutes the information needed to solve the problem. Arrange the above information and find the solution.

**Question 5** Explain how we can ensure that the optimal solution to a linear programming problem does not change when we simultaneously change the objective function values of the decision variables.

## Section – B

**Attempt any 2 out of 3 questions from this section. Each question carries 10 marks.**

**Question1** Three inputs (number of employees, capital, and deposits), and two outputs (loans and investments) for some banks are given below. Capital is measured by the book value of fixed assets and premises, and deposits are measured by the sum of long-term and saving deposits. Similarly, loans include loans to individuals, real estate loans, and commercial and industrial loans. Investments are measured using the value of all securities, other than those held in a bank's trading accounts. Capital, deposits, loans and investments are measured in appropriate money units (US$).

| Banks | Loans | Investments | Employees | Capital | Deposits |
|---|---|---|---|---|---|
| 1 | 945 | 233 | 520 | 91 | 3457 |
| 2 | 85 | 20 | 43 | 8 | 299 |
| 3 | 1200 | 323 | 643 | 109 | 4203 |
| 4 | 12 | 4 | 21 | 1 | 40 |
| 5 | 43 | 8 | 19 | 3 | 198 |
| 6 | 249 | 40 | 112 | 19 | 892 |
| 7 | 546 | 98 | 286 | 25 | 1417 |
| 8 | 325 | 75 | 215 | 20 | 999 |
| 9 | 1513 | 387 | 680 | 121 | 4802 |

Write the input minimization DEA (CCR) programs for any of the DMU for estimating the efficiency of Bank. Explain the process of determining the bench mark.

**Question 2** Burn-Off, a manufacturer of diet drinks is planning to introduce a drink that will magically burn away fat. The drink is bit expensive but Burn-Off guarantees that a person using this diet plan will lose up to 50 pounds in just three weeks. The drink is made up of four "*mystery*" ingredients (which we will call A, B, C and D). The plan calls for a person to consume at least 36 ounce per day. Each of the four ingredients contains different levels of three chemical compounds (which we will call X, Y and Z). Health regulations mandate that dosage consumed per day should contain minimum prescribed levels of chemicals X and Y and should not exceed maximum prescribed levels for the third chemical Z. The composition of the four ingredients in terms of the chemical compounds (units per ounce) is shown below along with the unit cost of prices of the ingredients. Burn-Off wants to find the optimal way to mix the ingredients to create the drink, at minimum cost per daily dose. Decision variables: Let A, B, C, and D denote the number of ounces of ingredients A, B, C, and D to use, respectively.

| Units of Chemical per Ounce of Ingredient | | | | | |
|---|---|---|---|---|---|
| | Ingredient | | | | |
| Chemical | A | B | C | D | Requirement |
| X | 3 | 4 | 8 | 10 | ≥ 280 units |
| Y | 5 | 3 | 6 | 6 | ≥ 200 units |
| Z | 10 | 25 | 20 | 40 | ≤ 1050 units |

| $ per ounce of ingredient | | | |
|---|---|---|---|
| 0.40 | 0.20 | 0.60 | 0.30 |

*The computer outputs of the sensitivity report for the problem are as follows.*

Adjustable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$3 | DVs A | 10.25 | 0 | 0.4 | 0.061111111 | 0.25 |
| $C$3 | DVs B | 0 | 0.06875 | 0.2 | 1E+30 | 0.06875 |
| $D$3 | DVs C | 4.125 | 0 | 0.6 | 1.5 | 0.073333333 |
| $E$3 | DVs D | 21.625 | 0 | 0.3 | 0.084615385 | 1E+30 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $F$5 | min req LHS | 36 | 0.375 | 36 | 16.5 | 1.277777778 |
| $F$6 | X LHS | 280 | 0.0875 | 280 | 41 | 11 |
| $F$7 | Y LHS | 205.75 | 0 | 200 | 5.75 | 1E+30 |
| $F$8 | Z LHS | 1050 | -0.02375 | 1050 | 47.14285714 | 346 |

a. Formulate LP as minimization of cost.

Based on the sensitivity report answer the followings:

b. What is the impact on cost if Burn-Off insists on using 1 ounce of ingredients B to make the drink?

c. There is some uncertainty in the cost of ingredient C. How sensitive is the current optimal solution?
d. Burn-Off can decrease the minimum requirement for chemical X by 5 units (from 280 to 275) provided the maximum limit allowed for chemical Z is reduced to 1000 units (that is, reduced by 50 units). Is this trade-off cost-effective for Burn-Off to implement?

**Question 3** First American Bank issues five types of loans. In addition, to diversify its portfolio, and to minimize risk, the bank invests in risk-free securities. The loans and the risk-free securities with their annual rate of return are given in the following Table.

**Table: Rates of Return for Financial Planning Problem**

| **Type of Loan or Security** | **Annual Rate of Return (%)** |
|---|---|
| Home Mortgage (first) | 6 |
| Home Mortgage (second) | 8 |
| Commercial loan | 11 |
| Automobile loan | 9 |
| Home improvement loan | 10 |
| Risk-free Securities | 4 |

The bank`s objective is to maximize the annual rate of return on investments subject to the following policies, restrictions and regulations:

1. The bank has $90 million in available funds.
2. Risk-free securities must contain at least 10 percent of the total funds available for investments.
3. Home improvement loans cannot exceed $8,000,000.
4. The investment in mortgage loans must be at least 60 percent of all the funds invested in loans.
5. The investment in first mortgage loans must be at least twice as much as the investment in second mortgage loans.
6. Home improvement loans cannot exceed 40 percent of the funds invested in first mortgage loans.
7. Automobiles loans and home improvement loans together may not exceed the commercial loans.
8. Commercial loans cannot exceed 50 percent of the total funds invested in mortgage loans.

## Section - C

**Compulsory Case Study (15 Marks)**

A new author sets three criteria for selecting a publisher for an OR text book: royalty percentage (R), marketing (M) and advance payment (A). Two publishers H and P have expressed interest in the book. Using the following comparison matrix, rank the two publishers and asses the consistency of the decision matrix when they are not consistent..

$$A = \begin{array}{c} \\ R \\ M \\ A \end{array} \begin{array}{c} \begin{array}{ccc} R & M & A \end{array} \\ \begin{pmatrix} 1 & 1 & \frac{1}{4} \\ 1 & 1 & \frac{1}{5} \\ 4 & 5 & 1 \end{pmatrix} \end{array}$$

$$A_R = \begin{array}{c} \\ H \\ P \end{array} \begin{array}{c} \begin{array}{cc} H & P \end{array} \\ \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & 1 \end{pmatrix} \end{array} \qquad A_M = \begin{array}{c} \\ H \\ P \end{array} \begin{array}{c} \begin{array}{cc} H & P \end{array} \\ \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & 1 \end{pmatrix} \end{array} \qquad A_A = \begin{array}{c} \\ H \\ P \end{array} \begin{array}{c} \begin{array}{cc} H & P \end{array} \\ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array}$$