PGDM (IBM), Batch 2014-16

Quantitative Techniques

INS-105

Trimester-I, End-Term Examination, September, 2014

Time Allowed: 2Hrs. & 30Mins.

SECTION-A **(Marks: 3X5)**

Q.1 Explain the difference between sampling error and non sampling error. Which type of error is more serious & why?

Q.2 Define'random variable'. How do you distinguish between discrete and continuous variables? Illustrate your answer with suitable examples.

Q.3 The latest nationwide political poll indicates that for Americans who are randomly selected, the probability that they are conservative is 0.55, the probability that they are liberal is 0.30, and the probability that they are middle-of-the-art is 0.15.Assuming that these probabilities are accurate, answer the following questions pertaining to a randomly chosen group of 10 Americans. (Table is not required).

(a) What is the probability that four are liberal?

(b) What is the probability that at least eight are liberal?

Q.4 A statistics instructor has observed that the number of typographical errors in new editions of text books varies considerably from book to book. After some analysis, he concluded that the number of errors is Poisson distributed with mean of 1.5 per 100 per pages. Suppose that the instructor has just received a copy of a new statistic book. He notices that there are 400 pages.

(a) What is the probability that there are no typos?

(b) What is the probability that there are five or fewer typos?

Q.5 As part of the quality assurance program, the Automobile Battery Company conducts tests on battery life. For a particular D-cell alkaline battery, the mean life is 19 hours. The useful life of the battery follows a normal distribution with a standard deviation of 1.2 hours. Answer the following questions.

(a) About 68 percent of the batteries failed between what two values?

(b) About 95 percent of the batteries failed between what two values?

(c) Virtually all of the batteries failed between what two values?

SECTION-B **(Marks:2X10)**

Q.6 Clara Voyant, whose job is predicting the future for her venture capital company, has just received the statistics describing her company's performance on 1,800 investments last year. Clara knows that, in general, investments generate profits that have a normal distribution with mean $7,500 and standard deviation $3,300.Even before she looked at the specific results from each of the 1,800 investments from last year; Clara was able to make some accurate predictions by using her knowledge of sampling distributions. Follow her analysis by finding the probability that the sample mean of last year's investments

(a) Exceeded $7,700.

(b) Was less than $7,400.

(c) Was greater than $7, 275, but less than $7,650?

Q.7 The McFarland Insurance Company Claim Department reports the mean cost to process a claim is $60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the claim department selected a random sample of 26 claims processed last months. The sample information is reported below:

$45	$49	$62	$40	$43	$61
48	53	67	63	78	64
48	54	51	56	63	69
58	51	58	59	56	57
38	76				

Q.8 IQ tests are designed to yield results that are approximately normally distributed. Researchers think that the population standard deviation is 15. A reporter is interested in estimating the average IQ of employees in a large high-tech firm in California. She gathers the IQ information on 22 employees of this firm and records the sample mean IQ is 106.

(a) Compute 90% and 99% confidence intervals of the average IQ in this firm.

(b) Use these results to infer if the mean IQ in this firm is significantly different from the national average of 100.

SECTION-C **(15 Marks)**

Case Study

Technological advances helped make inflatable paddle craft suitable for backcountry use. These blow-up rubber boats, which can be rolled into a bundle not much bigger than a golf bag, are large enough to accommodate one or two paddlers and their camping gear. Canoe & Kayak magazine tested boats from nine manufacturers to determine how they would perform on a three-day wilderness paddling trip. One of the criteria in their evaluation was the baggage capacity of the boat, evaluated using a 4-point rating scale from 1 (lowest rating) to 4 (highest rating). The following data show the baggage capacity rating and the price of the boat.

Boat	Baggage Capacity	Price ($)
S14	4	1595
Orinoco	4	1399
Outside Pro	4	1890
Explorer 380X	3	795
River XK2	2.5	600
Sea Tiger	4	1995
Maverick II	3	1205
Starlite 100	2	583
Fat Pack Cat	3	1048

Questions:

a. Develop a scatter diagram for these data with baggage capacity rating as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between baggage capacity and price/
c. Draw a straight line through the data to approximate a liner relationship between baggage capacity and price.
d. Use the least squares method to develop the estimated regression equation.
e. Provide an interpretation for the slope of the estimated regression equation.

Formulae:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$P\left[\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right] = (1-\alpha)$$

$$\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$$\text{Probability of } x \text{ success in } n \text{ trials} = P(x) = \frac{!n}{!(n-x)!(x)} p^x q^{n-x}$$

Mean and variance of a binomial probability distribution

$$\text{Mean} = \mu = E(x) = np$$

$$\text{Var }(x) = \sigma^2 = np(1-p) = npq$$

$$\text{Standard deviation} = \sigma - \sqrt{npq}$$

Poisson formula

$$P(x) = \frac{\lambda^x \times e^{-\lambda}}{!x}$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{[\sum (x - \bar{x})^2][\sum (y - \bar{y})^2]}}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

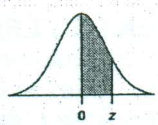
$$b_1 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

APPENDIX C-1

STANDARD NORMAL AREAS

Example: $P(0 < z < 1.96) = .4750$

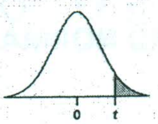


This table shows the normal area between 0 and z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.49865	.49869	.49874	.49878	.49882	.49886	.49889	.49893	.49896	.49900
3.1	.49903	.49906	.49910	.49913	.49916	.49918	.49921	.49924	.49926	.49929
3.2	.49931	.49934	.49936	.49938	.49940	.49942	.49944	.49946	.49948	.49950
3.3	.49952	.49953	.49955	.49957	.49958	.49960	.49961	.49962	.49964	.49965
3.4	.49966	.49968	.49969	.49970	.49971	.49972	.49973	.49974	.49975	.49976
3.5	.49977	.49978	.49978	.49979	.49980	.49981	.49981	.49982	.49983	.49983
3.6	.49984	.49985	.49985	.49986	.49986	.49987	.49987	.49988	.49988	.49989
3.7	.49989	.49990	.49990	.49990	.49991	.49991	.49992	.49992	.49992	.49992

APPENDIX D

STUDENT'S *t* CRITICAL VALUES



This table shows the *t*-value that defines the area for the stated degrees of freedom (ν).

	Confidence Level						*Confidence Level*				
	.80	.90	.95	.98	.99		.80	.90	.95	.98	.99
	Significance Level for Two-Tailed Test						*Significance Level for Two-Tailed Test*				
	.20	.10	.05	.02	.01		.20	.10	.05	.02	.01
	Significance Level for One-Tailed Test						*Significance Level for One-Tailed Test*				
ν	.10	.05	.025	.01	.005	ν	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.656	36	1.306	1.688	2.028	2.434	2.719
2	1.886	2.920	4.303	6.965	9.925	37	1.305	1.687	2.026	2.431	2.715
3	1.638	2.353	3.182	4.541	5.841	38	1.304	1.686	2.024	2.429	2.712
4	1.533	2.132	2.776	3.747	4.604	39	1.304	1.685	2.023	2.426	2.708
5	1.476	2.015	2.571	3.365	4.032	40	1.303	1.684	2.021	2.423	2.704
6	1.440	1.943	2.447	3.143	3.707	41	1.303	1.683	2.020	2.421	2.701
7	1.415	1.895	2.365	2.998	3.499	42	1.302	1.682	2.018	2.418	2.698
8	1.397	1.860	2.306	2.896	3.355	43	1.302	1.681	2.017	2.416	2.695
9	1.383	1.833	2.262	2.821	3.250	44	1.301	1.680	2.015	2.414	2.692
10	1.372	1.812	2.228	2.764	3.169	45	1.301	1.679	2.014	2.412	2.690
11	1.363	1.796	2.201	2.718	3.106	46	1.300	1.679	2.013	2.410	2.687
12	1.356	1.782	2.179	2.681	3.055	47	1.300	1.678	2.012	2.408	2.685
13	1.350	1.771	2.160	2.650	3.012	48	1.299	1.677	2.011	2.407	2.682
14	1.345	1.761	2.145	2.624	2.977	49	1.299	1.677	2.010	2.405	2.680
15	1.341	1.753	2.131	2.602	2.947	50	1.299	1.676	2.009	2.403	2.678
16	1.337	1.746	2.120	2.583	2.921	55	1.297	1.673	2.004	2.396	2.668
17	1.333	1.740	2.110	2.567	2.898	60	1.296	1.671	2.000	2.390	2.660
18	1.330	1.734	2.101	2.552	2.878	65	1.295	1.669	1.997	2.385	2.654
19	1.328	1.729	2.093	2.539	2.861	70	1.294	1.667	1.994	2.381	2.648
20	1.325	1.725	2.086	2.528	2.845	75	1.293	1.665	1.992	2.377	2.643
21	1.323	1.721	2.080	2.518	2.831	80	1.292	1.664	1.990	2.374	2.639
22	1.321	1.717	2.074	2.508	2.819	85	1.292	1.663	1.988	2.371	2.635
23	1.319	1.714	2.069	2.500	2.807	90	1.291	1.662	1.987	2.368	2.632
24	1.318	1.711	2.064	2.492	2.797	95	1.291	1.661	1.985	2.366	2.629
25	1.316	1.708	2.060	2.485	2.787	100	1.290	1.660	1.984	2.364	2.626
26	1.315	1.706	2.056	2.479	2.779	110	1.289	1.659	1.982	2.361	2.621
27	1.314	1.703	2.052	2.473	2.771	120	1.289	1.658	1.980	2.358	2.617
28	1.313	1.701	2.048	2.467	2.763	130	1.288	1.657	1.978	2.355	2.614
29	1.311	1.699	2.045	2.462	2.756	140	1.288	1.656	1.977	2.353	2.611
30	1.310	1.697	2.042	2.457	2.750	150	1.287	1.655	1.976	2.351	2.609
31	1.309	1.696	2.040	2.453	2.744	∞	1.282	1.645	1.960	2.326	2.576
32	1.309	1.694	2.037	2.449	2.738						
33	1.308	1.692	2.035	2.445	2.733						
34	1.307	1.691	2.032	2.441	2.728						
35	1.306	1.690	2.030	2.438	2.724						

Note: As n increases, critical values of Student's t approach the z-values in the last line of this table. A common rule of thumb is to use z when $n > 30$, but that is *not* conservative.