FORECASTING OF CBOE VOLATILITY OF VIX (VVIX) AS A MEASURE OF PREDICTING EXTREME EVENTS – AN ARIMA FRAMEWORK

Arindam Banerjee

Abstract— Volatility of VIX or VVIX is an indicator that is based upon volatility index or VIX. Volatility index or VIX is also known as 'investor fear gauge'. It is an indicator that may be considered as a strong indicator of investors' fears and emotions (Durand et al., 2011; Whaley, 2009). It measures investors' view of the market's volatility in the immediate term. The VVIX Index is on the other hand, an indicator of the expected volatility of the 30-day forward price of the VIX. VVIX may provide a further insight into the investors' expectations about future volatility in the market (Nikkinen & Peltomäki, 2019). VVIX is an interesting market indicator that provides an indication towards future tail risk (Park, 2015). The objective of this study is to offer a unique and simple method of forecasting VVIX. Our argument is, forecasting of VVIX may help market participants in gauging the possibility of tail risk, and may lead to better investment decision.

Index Terms— Auto Regressive Integrated Moving Average (ARIMA), Extreme event**,** Forecasting, Tail risk, **V**VIX,

1 INTRODUCTION

Volatility of volatility index or VVIX captures the expected volatility of VIX. According to Park (2015), VVIX may be a good tail risk indicator. Tail risk may be defined as extraordinary risk that may fall beyond the +3 standard deviation. Most of the financial models e.g. the modern portfolio theory, Black and Scholes derivative pricing model etc. assumes a normality of sample as a basic assumption. A normal distribution curve assumes that, given enough observations, all values in the sample will be distributed equally above and below the mean. About 99.7% of all variations falls within three standard deviations of the mean and therefore there is only a 0.3% chance of an extremeeventoccurring (https://www.nasdaq.com/articles/fat-tail-risk-what-it-means-and-why-you-should-be-aware-it-2015-11-02). However, in practice there are many instances of extreme event happening in stock markets across the world. The US subprime crisis (2008) is probably the most significant instance in recent times. Park (2015) demonstrated in his seminal paper, that VVIX may prove to be a useful tool for predicting tail risk.

Our argument is, forecasting of VVIX may help the market participants in gauging the possibility of tail risk or extreme events, and may lead to better investment decisions. This paper develops an analytical model for forecasting VVIX in the auto-regressive integrated moving average (ARIMA) framework for the period starting from March 2009 to October 2016. In this context, our first contribution to the literature is methodology. To prove the robustness of our model, it is validated by using daily data from November 2016 to October 2017.This study uses CBOE VVIX, which is traded at the Chicago Board of Exchange (CBOE) platform, USA. VVIX is a volatility of volatility index launched by CBOE, which measures the expected volatility of CBOE volatility index (VIX). The motivation of this study lies not only on the widespread agreement that VVIX is a good predictor of fat tail risks (Park, 2015), but also on the fact that there are many trading strategies that rely on the VVIX index for speculative and hedging purposes. It is important to being aware of possibility of extreme events. Also adopting an adequate hedging strategy for addressing possible tail risk events is also important to protect investments from economic turmoil. The ideal portfolio should not only generate good return for each unit of volatility, but also should be able to protect itself from tail risk. A good model for predicting VVIX can prove itself useful in this regard.Since the CBOE VVIX is a good predictor of extreme events in near future, so by forecasting the future value of CBOE VVIX, we may create a better trading strategy to avoid losses. The objective of this study is to fit a forecasting model on CBOE VVIX using ARIMA. The model would be useful predicting future movement of volatility and expected extreme events.VVIX is also less prone to measurement error. VVIX is in a sense the volatility predicted by market participants. (Płuciennik, Buszkowska, 2006). With the advent of volatility based indices, it is important to develop a model that may make a reliable forecast of VVIX, as a predictor of 'fat tail risk'.The rest of the paper is organized as follows. Section 'objective' describes the motivation behind the study. Section 'period of study and data' underlines the period covered under this study for fitting this model and also for validating the same. The section 'description of methodology' presents the description about methodology used in this paper. The section 'empirical results and analysis' shows the data analysis with the estimated results. The section 'evaluation of forecasts' presents the validation of the model. And finally, the

- *Arindam Banerjee is currently working as Asst. Professor at Birla Institute of Management Technology, Greater Noida. India. E-mail: arindam20011@gmail.com*

paper concludes with the section 'conclusion and scope', which also indicates towards few areas for further research.

2 OBJECTIVE

The objective of this study is to develop a forecasting model to predict CBOE VVIX. We apply auto-regressive integrated moving average (ARIMA) technique for developing the model.

3. PERIOD OF STUDY

The period of study under consideration is from March 2009 until October 2016 for fitting the model. We also used daily data from November 2016 to October 2017 for validating the ARIMA model. The daily closing value of VVIX is downloaded from the website of the Chicago Board of Exchange (www.cboe.com).

4. DATA AND METHODOLOGY

This section describes the methodology used for designing the VVIX model that forecasts the future direction. For this purpose, the ARIMA model has been used. But before that the unit root test has been performed to check the stationarity of the dataset.

4.1 Unit root analysis

As many a times, the time series variables suffer from the non-stationary problem, we have tested for unit root under augmented Dickey-Fuller (ADF) test. Section 5.1 shows the result and analysis of ADF test.

4.2 ARIMA

The basic idea behind ARIMA or the Box-Jenkins (BJ) methodology for forecasting is to analyse the probabilistic or stochastic properties of economic time series on their own under the philosophy "let the data speak for themselves." This concept is very different from traditional regression models, in which the dependent variable Y_t is explained by k explanatory variables X_1, X_2,…………X_n, the BJ time series models allow Y_t to be explained by the past, or lagged, values of Y_t itself and the current and lagged values of u_t, which is an uncorrelated random error with zero mean and constant variance σ^2 – that is, a white noise error term. The BJ methodology is based on the assumption that the time series under consideration is stationary.

4.2.1 The AR model

Consider the following model:

$$Y_t=B_0+B_1Y_{t-1}+B_2Y_{t-2}+\ldots\ldots.+B_pY_{t-p}+u_t$$

Where u_t is a white noise error term.

This model is termed as an AR model of order p, AR (p), for it involves regressing Y at time t on its values lagged p periods into the past, the value of p being determined empirically using some criterion, such as the Akaike information criterion.

4.2.2 The MA model

The AR process is not the only mechanism that may have generated Y_t. In some situation, it might be possible to capture the process of generation of Y_t series by following model.

$$Y_t=u_t+\Theta u_{t-1}$$

Where, as before, u_t is a white error term. The model implies that Y_t is determined as a MA of the current and immediate past values of the error term. This model is called the first-order MA or MA(1) model.

The general form of the MA model is an MA(q) model of the form

$$Y_t=u_t+\Theta_1u_{t-1}+\Theta_2u_{t-2}+\ldots\ldots.+\Theta_qu_{t-q}$$

It appears that a MA process is simply a linear combination of white noise processes, so that Y_t depends on the current and previous values of a white noise error term. Further, as long as q is finite, the MA(q) process is stationary as it is an average of q stationary white noise error terms which are stationary.

4.2.3 The ARMA model

If we suppose that Y_t has characteristics of both AR and MA, then it is called ARMA process. For example, an ARMA (1,1) model may be written as

$$Y_t=\Phi Y_{t-1}+u_t+\Theta u_{t-1}$$

In general, an ARMA (p,q) process will have p AR and q MA terms. It is written as

$$Y_t=\Phi_1Y_{t-1}+\Phi_2Y_{t-2}+\ldots\ldots+\Phi_pY_{t-p}+u_t+\Theta_1u_{t-1}+\Theta_2u_{t-2}+\ldots\ldots+\Theta_qu_{t-q}$$

4.2.4 The ARIMA model

If a time series is integrated of order d and we apply ARMA (p,q) model to it, then we say that the original time series is ARIMA (p,d,q), i.e., it is an ARIMA time series. Clearly, if a time series is ARIMA (2,1,2), it has to be differenced once to make it stationary and the stationary time series can be modelled as ARMA (2,2) process, i.e., it will have two AR and two MA terms. Similarly, an ARIMA (p,0,p) series is same as ARMA (p,q) when the time series is stationary at the beginning. On the other hand, ARIMA (p,0,0) and ARIMA (0,0,q) series represent AR (p) and MA (q) stationary processes, respectively. Thus, given the values of p,d, and q, one can say what process is being modelled.

5. EMPIRICAL RESULTS AND ANALYSIS

This study fits a forecasting model based on CBOE VVIX, the indicator of extreme events. The study considers log of daily VVIX (LVVIX) value. The idea behind plotting the log of a variable represents a relative change (or rate of return), whereas a change in the log of a variable itself represents an absolute change. Returns are unit free and they are comparable (Gujarati, 2015). The total number of observations are n=616.

5.1 Test of Stationarity

First test of stationarity of the time series data is tasted. To test stationarity, the ADF test is being used. The test is performed by using the following form:

$$\Delta LVVIX_t = B_1 + B_2 t + B_3 LVVIX_{t-1} + \sum_{i=1}^{m} \alpha_i LVVIX_{t-1} + \varepsilon_t \qquad (1)$$

In each case, the null hypothesis is $B_3 = 0$ (i.e. unit root exists) and the alternative hypothesis is that $B_3 < 0$ (i.e. no unit root). The result of the unit root test of LVVIX with intercept is shown in table 1.

Table 1: Unit root test of VVIX with intercept

Null Hypothesis: LN_VVIX_ has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.600639	0.0000
Test critical values:	1% level	-3.973046	
	5% level	-3.417142	
	10% level	-3.130953	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LN_VVIX_)
Method: Least Squares
Date: 12/24/19 Time: 15:03
Sample (adjusted): 1/08/2007 10/15/2018
Included observations: 615 after adjustments

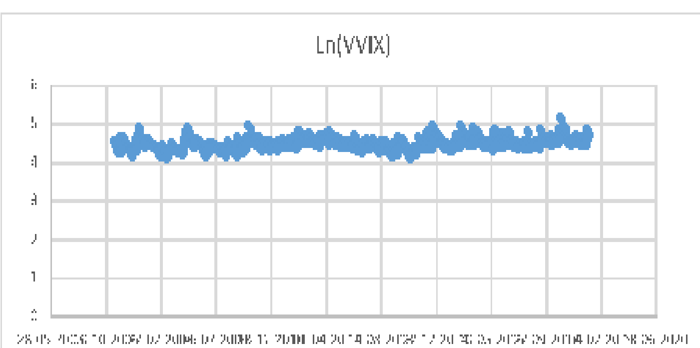
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LN_VVIX_(-1)	-0.262920	0.027386	-9.600639	0.0000
C	1.156340	0.120808	9.571724	0.0000
@TREND("1/01/2007")	6.68E-05	2.30E-05	2.907163	0.0038

R-squared	0.130951	Mean dependent var	0.000395
Adjusted R-squared	0.128111	S.D. dependent var	0.103886
S.E. of regression	0.097003	Akaike info criterion	-1.823278
Sum squared resid	5.758695	Schwarz criterion	-1.801709
Log likelihood	563.6581	Hannan-Quinn criter.	-1.814891
F-statistic	46.10889	Durbin-Watson stat	2.092748
Prob(F-statistic)	0.000000		

As the R^2 (0.130951) is less than Durbin-Watson stat (2.092748), therefore the regression is not spurious (Bhowmick, 2015).

Table 1 shows the results of ADF test. The LVVIX lagged one period. The ADF test statistic is (-9.600639). However, the DF critical values are -3.973046 (1% level), -3.417142 (5% level), and -3.130953 (10% level). In absolute terms, 9.600639 is greater than any of the DF critical t values in absolute terms. Hence, the conclusion is that the VVIX time series is stationary (Gujarati, 2015). To confirm, also plotted the graph of LVVIX over time (Figure 1). The graph confirms the stationarity of LVVIX.

Figure 1: LVVIX time series



5.2 Determination of p,q, and d

As the LVVIX is stationary time series with level unit root, therefore we consider the value of d=0. We already have showed that the level order time series LVVIX is stationary. So, we work with LVVIX only here. To see, which ARIMA model fits LVVIX, and following the BJ methodology, we computed the correlogram of this series upto 36 lags. For determining the number of lags, we followed the rule of thumb suggested by Schwert (1989). Due to space constraint, we show the correlogram upto 15 lags in Table 2 below. The complete correlogram is given in annexure A1 at the end of this paper.

Table 2: AC function and PAC function of LVVIX

Date: 12/24/19 Time: 15:28
Sample: 1/01/2007 10/15/2018
Included observations: 616

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|***** |</td><td>.|***** |</td><td>1</td><td>0.755</td><td>0.755</td><td>353.28</td><td>0.000</td></tr>
<tr><td>.|**** |</td><td>.|. |</td><td>2</td><td>0.600</td><td>0.068</td><td>576.58</td><td>0.000</td></tr>
<tr><td>.|**** |</td><td>.|. |</td><td>3</td><td>0.502</td><td>0.066</td><td>733.31</td><td>0.000</td></tr>
<tr><td>.|*** |</td><td>.|* |</td><td>4</td><td>0.456</td><td>0.098</td><td>862.67</td><td>0.000</td></tr>
<tr><td>.|*** |</td><td>.|. |</td><td>5</td><td>0.385</td><td>-0.032</td><td>955.02</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>6</td><td>0.323</td><td>-0.002</td><td>1020.1</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>7</td><td>0.298</td><td>0.062</td><td>1075.6</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>8</td><td>0.281</td><td>0.028</td><td>1125.0</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>9</td><td>0.245</td><td>-0.019</td><td>1162.7</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|. |</td><td>10</td><td>0.194</td><td>-0.041</td><td>1186.4</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>11</td><td>0.201</td><td>0.092</td><td>1211.8</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>12</td><td>0.223</td><td>0.067</td><td>1243.1</td><td>0.000</td></tr>
<tr><td>.|** |</td><td>.|. |</td><td>13</td><td>0.242</td><td>0.058</td><td>1280.0</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>*|. |</td><td>14</td><td>0.200</td><td>-0.075</td><td>1305.3</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|. |</td><td>15</td><td>0.188</td><td>0.027</td><td>1327.8</td><td>0.000</td></tr>
</table>

Table 2 produces two types of correlation coefficients: Autocorrelation (AC) and partial AC (PAC). The AC function (ACF) shows correlation of LVVIX with its values with various lags. The PAC function (PACF) shows the correlation between observations that are k periods apart after controlling for the effects of intermediate lags. The BJ methodology uses both these correlation coefficients to identify the type of ARMA model that is appropriate for this case.

Table 2 shows gradual decline of AC and changes in positive and negative signs of PAC. However, it does not show any sign of exponential decay for any sustained period.

To see, which correlations are statistically significant, we calculate the standard error of sample correlation coefficients given by $\sqrt{1/n} = \sqrt{1/616} = 0.040291$, where n is the sample size. Therefore, the 95% confidence interval for the true correlation coefficients is about $0 \pm 1.96\ (0.040291) = (-0.07897$ to $0.07897)$. Correlation coefficients lying outside these bounds are statistically significant at 5% level. On this basis, it seems that PACF correlations at lag(s) 1, 4, 11, 26 are statistically significant.

Since we do not have any clear-cut pattern of the ACF and PACF, we will proceed by trial and error.

First, we fit an AR model at lags 1, 4, 11, 26.

Then we fit an MA model at lags 1, 4, 11, and 26. The results of AR (1,4,11,26) is shown in Table 3.

Table 3: AR model fit at lags 1, 4, 11, and 26

Dependent Variable: LN_VVIX_
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/24/19 Time: 17:09
Sample: 1/01/2007 10/15/2018
Included observations: 616
Convergence achieved after 12 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.478506	0.031811	140.7834	0.0000
AR(1)	0.705154	0.026955	26.16000	0.0000
AR(4)	0.087556	0.032379	2.704051	0.0070
AR(11)	0.035300	0.030393	1.161437	0.2459
AR(26)	0.027778	0.025998	1.068467	0.2857
SIGMASQ	0.009261	0.000392	23.62167	0.0000

R-squared	0.583546	Mean dependent var	4.475200
Adjusted R-squared	0.580133	S.D. dependent var	0.149247
S.E. of regression	0.096708	Akaike info criterion	-1.822926
Sum squared resid	5.704961	Schwarz criterion	-1.779843
Log likelihood	567.4613	Hannan-Quinn criter.	-1.806174
F-statistic	170.9498	Durbin-Watson stat	2.017966
Prob(F-statistic)	0.000000		

Since the AR(11) and AR(26) coefficients are not significant, we can drop these from consideration and re-estimate the model with AR (1) and AR (4). The result is shown in table 4 below.

Table 4: AR(1) and AR(4) model for LVVIX

Dependent Variable: LN_VVIX_
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/24/19 Time: 17:22
Sample: 1/01/2007 10/15/2018
Included observations: 616
Convergence achieved after 10 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.477465	0.024002	186.5462	0.0000
AR(1)	0.706885	0.027091	26.09262	0.0000
AR(4)	0.101255	0.028943	3.498440	0.0005
SIGMASQ	0.009310	0.000394	23.62713	0.0000

R-squared	0.581370	Mean dependent var		4.475200
Adjusted R-squared	0.579318	S.D. dependent var		0.149247
S.E. of regression	0.096802	Akaike info criterion		-1.824349
Sum squared resid	5.734781	Schwarz criterion		-1.795626
Log likelihood	565.8995	Hannan-Quinn criter.		-1.813181
F-statistic	283.3034	Durbin-Watson stat		2.022257
Prob(F-statistic)	0.000000			
Inverted AR Roots	.86	.14+.49i	.14-.49i	-.44

In the next stage, we fit the MA model. Again, we go through the trial and error method for MA(1), MA(4), MA(11) and MA(26). The result of MA model is given in Table 5 below.
The model is significant at MA (1) and MA (4).

Table 5: MA model fit at lags 1,4,11,26

Dependent Variable: LN_VVIX_
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/24/19 Time: 17:31
Sample: 1/01/2007 10/15/2018
Included observations: 616
Convergence achieved after 26 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.475554	0.009023	496.0396	0.0000
MA(1)	0.570309	0.029607	19.26296	0.0000
MA(4)	0.185465	0.034655	5.351718	0.0000
MA(11)	0.051046	0.037796	1.350590	0.1773
MA(26)	0.036483	0.031429	1.160809	0.2462
SIGMASQ	0.012529	0.000611	20.49319	0.0000

R-squared	0.436586	Mean dependent var	4.475200
Adjusted R-squared	0.431968	S.D. dependent var	0.149247
S.E. of regression	0.112484	Akaike info criterion	-1.521203
Sum squared resid	7.718161	Schwarz criterion	-1.478120
Log likelihood	474.5306	Hannan-Quinn criter.	-1.504452
F-statistic	94.53697	Durbin-Watson stat	1.507357
Prob(F-statistic)	0.000000		

Thus we use ARIMA (1,0,1), ARIMA (1,0,4), ARIMA (4,0,1) and ARIMA (4,0,4).

Table 6 shows ARIMA (1,0,1) model fit.

Table 6: ARIMA (1,0,1) model fit

Dependent Variable: LN_VVIX_
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/24/19 Time: 17:44
Sample: 1/01/2007 10/15/2018
Included observations: 616
Convergence achieved after 12 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.477141	0.021126	211.9230	0.0000
AR(1)	0.809241	0.032594	24.82756	0.0000
MA(1)	-0.120894	0.052698	-2.294120	0.0221
SIGMASQ	0.009414	0.000395	23.80936	0.0000

R-squared	0.576680	Mean dependent var	4.475200
Adjusted R-squared	0.574605	S.D. dependent var	0.149247
S.E. of regression	0.097342	Akaike info criterion	-1.813276
Sum squared resid	5.799025	Schwarz criterion	-1.784554
Log likelihood	562.4891	Hannan-Quinn criter.	-1.802109
F-statistic	277.9048	Durbin-Watson stat	1.983480
Prob(F-statistic)	0.000000		

In table 6, both the AR(1) and MA(1) is significant. So, we will consider ARIMA (1,0,1) for fitting the model.
Next, we try ARIMA (1,0,4) for fitting the model fit.

Table 7: ARIMA (1,0,4) model fit

Dependent Variable: LN_VVIX_
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 01/02/20 Time: 11:36
Sample: 1/01/2007 10/15/2018
Included observations: 616
Convergence achieved after 10 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.476882	0.018740	238.8958	0.0000
AR(1)	0.737267	0.026013	28.34204	0.0000
MA(4)	0.103093	0.039804	2.590031	0.0098
SIGMASQ	0.009375	0.000395	23.71398	0.0000

R-squared	0.578436	Mean dependent var		4.475200
Adjusted R-squared	0.576369	S.D. dependent var		0.149247
S.E. of regression	0.097140	Akaike info criterion		-1.817410
Sum squared resid	5.774972	Schwarz criterion		-1.788687
Log likelihood	563.7621	Hannan-Quinn criter.		-1.806242
F-statistic	279.9120	Durbin-Watson stat		2.072121
Prob(F-statistic)	0.000000			
Inverted AR Roots	.74			
Inverted MA Roots	.40-.40i	.40-.40i	-.40+.40i	-.40+.40i

From table 7, both AR(1) and MA(4) is both are significant. So, we will accept ARIMA (1,0,4). Table 8 shows ARIMA (4,0,1) results, in which both AR(4) and MA(1) is significant. So we may accept ARIMA (4,0,1) also.

Table 8: ARIMA (4,0,1) model fit

Dependent Variable: LN_VVIX_
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 01/02/20 Time: 13:27
Sample: 1/01/2007 10/15/2018
Included observations: 616
Convergence achieved after 12 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.476031	0.012356	362.2445	0.0000
AR(4)	0.367070	0.038749	9.472948	0.0000
MA(1)	0.584715	0.030563	19.13131	0.0000
SIGMASQ	0.011593	0.000538	21.56872	0.0000
R-squared	0.478688	Mean dependent var	4.475200	
Adjusted R-squared	0.476132	S.D. dependent var	0.149247	
S.E. of regression	0.108023	Akaike info criterion	-1.604996	
Sum squared resid	7.141414	Schwarz criterion	-1.576273	
Log likelihood	498.3386	Hannan-Quinn criter.	-1.593828	
F-statistic	187.3200	Durbin-Watson stat	1.675612	
Prob(F-statistic)	0.000000			
Inverted AR Roots	.78	.00+.78i	-.00-.78i	-.78
Inverted MA Roots	-.58			

Next, we consider the ARIMA (4,0,4). The result is given in table 9 below.

Table 9: ARIMA (4,0,4) model fit

Dependent Variable: LN_VVIX_
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 01/02/20 Time: 11:46
Sample: 1/01/2007 10/15/2018
Included observations: 616
Convergence achieved after 12 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.476282	0.014407	310.7119	0.0000
AR(4)	0.687677	0.064834	10.60680	0.0000
MA(4)	-0.298982	0.083637	-3.574752	0.0004
SIGMASQ	0.017300	0.000826	20.93520	0.0000
R-squared	0.222056	Mean dependent var	4.475200	
Adjusted R-squared	0.218242	S.D. dependent var	0.149247	
S.E. of regression	0.131960	Akaike info criterion	-1.204390	
Sum squared resid	10.65699	Schwarz criterion	-1.175667	
Log likelihood	374.9520	Hannan-Quinn criter.	-1.193222	
F-statistic	58.22955	Durbin-Watson stat	0.697311	
Prob(F-statistic)	0.000000			
Inverted AR Roots	.91	.00-.91i	.00+.91i	-.91
Inverted MA Roots	.74	-.00+.74i	-.00-.74i	-.74

From table 9, it appears that ARIMA (4,0,4) is also suitable for the model. So, the question is out of these four models ARIMA models, which one we should select.

From the Akaike info criterion and Schwarz criterion, it may be observed that the it's minimum in case of ARIMA (1,0,4) model. It seems that the ARIMA (1,0,4) is the most appropriate model fit to depict the behavior of the level differences of the logs of daily closing VVIX over the sample period. After going through the abovementioned tests, and under consideration of principles of parsimony, finally, we select the ARIMA (1,0,4) or ARMA (1,4) as a fit model for forecasting of VVIX. This model may be used as an estimator for predicting the future values of VVIX.The generalized ARIMA (1,0,4) model may be written as (Chatfield, 2003):

$X_t = \mu(1-\alpha)+\alpha\ (x_{t-1})+\beta_1 e_{t-1}+\beta_2 e_{t-2}+\beta_3 e_{t-3}+\beta_4 e_{t-4}$

Or

$X_t = \mu(1-\alpha)+\alpha\ (x_{t-1})+\beta_1(x_{t-1} - x_{t-2})+\beta_2\ (x_{t-2} - x_{t-3})+\beta_3\ (x_{t-3} - x_{t-4})+\beta_4\ (x_{t-4} - x_{t-5})$ (2)

Now, we may put the values of ARIMA (1,0,4) from table 7 into equation 2, where

$\mu = 4.476882$
$\alpha = 0.737267$
$\beta_1 = 0$
$\beta_2 = 0$
$\beta_3 = 0$
$\beta_4 = 0.103093$

By putting the abovementioned values in equation 2, the model becomes,

$X_t = 4.476882\ (1-0.737267)+ 0.737267\ (x_{t-1})+ 0.103093\ (x_{t-4} - x_{t-5})$ (3)

Using this model, we compute the difference between the observed value and the computed model value for the sample data using equation 3, and find root mean squared errors (RMSE) equals to 0.097424, which again established the appropriateness of model.

5.3 ARIMA Forecasting

We now use ARIMA (1,0,4) model for forecasting VVIX. Figure 2 shows the static forecast of VVIX. This figure shows the actual and forecast values of logs of closing VVIX, as well as the confidence interval of the forecast. The accompanying table gives the same measures of the quality of the forecast, namely RMSE, mean absolute error (MAE), mean absolute percent error, and Theil inequality coefficient. The Theil coefficient is very low (0.010880), suggesting that the fitted model is quite good. This is clearly shown in Figure 2, which demonstrates how closely the actual and forecast values track each other.

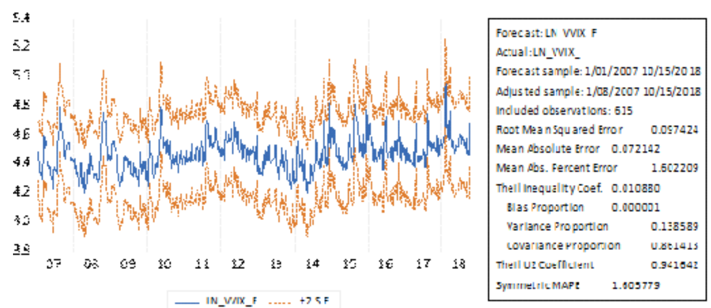


Figure 2: Static forecast of LVVIX

6. EVALUATION OF FORECASTS

The forecast of VVIX appears to be very reliable on the basis of the following criterion:

i. The estimated coefficients of both AR(1) and MA(4) terms are statistically significant.
ii. The value of RMSE for the estimated ARIMA (1,0,4) model is 0.097424, which is pretty low.
iii. The values of 'bias proportion', 'variance proportion' and 'covariance proportion' are 0.000001, 0.138589,

and 0.861413. Since the values of 'bias proportion', and 'variance proportion' is low and the 'covariance proportion' is high, therefore the forecast may be considered satisfactory.

iv. All inverted AR and MA roots are within the unit circle (figure 3), which implies that the chosen ARIMA model is stationary and the model has been correctly specified.

For validation of the model, we again considered the daily VVIX data from 22nd October 2018 to 18th October 2019. Table 10 shows the beginning of month estimated value and the observed value of log of VVIX by using equation 3. Though all the computations are based on daily closing quotes of VVIX, in table10, we only showed the month-end data for eleven months due to limited space.

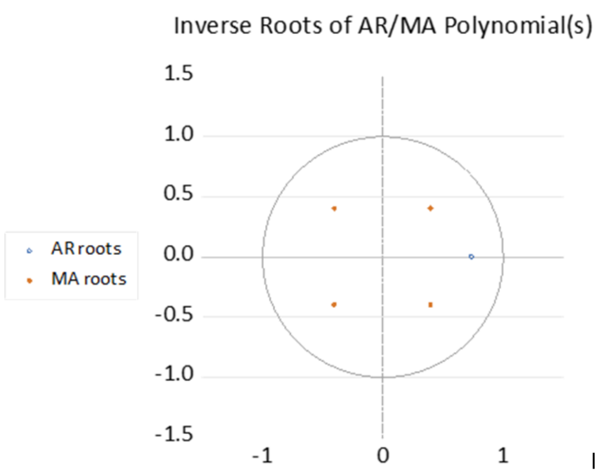


Figure 3: Inverse roots of autoregressive / moving average polynomials of VVIX

Table 10: Forecasted values of log of VVIX

Date	Observed Value	Model Value
03-12-2018	4.569439355	4.564811784
07-01-2019	4.429029464	4.504282961
04-02-2019	4.415219566	4.437177181
04-03-2019	4.468662918	4.363023227
01-04-2019	4.370712849	4.570657506
06-05-2019	4.564452374	4.456793932
03-06-2019	4.454347296	4.506866548
01-07-2019	4.382526522	4.42366394
05-08-2019	4.620255794	4.443080383
02-09-2019	4.547541094	4.650149278
07-10-2019	4.581901528	4.599983336

7. CONCLUSION AND FUTURE SCOPE

Our objective was to fit a forecasting model for VVIX. Based on literature (Park, 2015), we considered the VVIX as an indicator for tail risk i.e. possibility of extreme events. We find ARIMA (1,0,4) is the fittest model to forecast future VVIX values. The evaluation of forecasting ARIMA model is also found to be reliable. A reliable forecast of VVIX may prove to be very useful in predicting how the possibility of extreme event may emerge in near term. Investors may find this extremely useful in taking investment decisions. They will be able to take appropriate hedging decisions to protect the investment portfolio. This study may be extended by linking VVIX with the VIX returns. A derivative trader may be able to take a better decision by considering the forecasted values of VVIX.

REFERENCES

[1] Bhowmick, S (2015). Principles of Econometrics. New Delhi. Oxford University Press

[2] Chatfield C (2003), The Analysis of Time series. USA: Chapman and Hall

[3] Durand, R.B., Lim, D., Zumwalt, J.K. (2011), Fear and Fama-French factors. Financial Management, 40(2), 409-426

[4] Gujarati, N (2015), Econometrics by Example, USA: Palgrave Macmillan

[5] Jussi Nikkinen & Jarkko Peltomäki (2019): Crash Fears and Stock Market Effects: Evidence From Web Searches and Printed News Articles, Journal of Behavioral Finance, DOI: 10.1080/15427560.2019.1630125

[6] Park, Yang-Ho (2015), Volatility-of-volatility and tail risk hedging returns. Journal of Financial Markets, http://dx.doi.org/10.1016/j.finmar.2015.05.003

[7] Schwert, G. W. (1989), Why does stock market volatility change over time? The Journal of Finance, 44(5), 1115-1153

[8] Whaley, R.J. (2009), Understanding VIX. The Journal of Portfolio Management, 35, 98-105