

PGDM, 2021-23  
Statistics for Business Analysis  
DM-110

Trimester – I, End-Term Examination: October 2021

Time allowed: 1 Hour 30 Minutes  
Max Marks: 30

Roll No: \_\_\_\_\_

**Instruction:** Students are required to write Roll No on every page of the question paper. All instructions on the reverse of the admit card should be followed meticulously. Use of calculator is allowed.

**SECTION A – (5 marks \* 3 questions) = 15 Marks**

**(CILO 1)**

**A1a** Is your favorite TV program often interrupted by advertising? CNBC presented statistics on the average number of programming minutes in a half-hour sitcom (CNBC, February 23, 2006). The following data (in minutes) are representative of their findings.

|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| 21.06 | 22.24 | 20.62 | 21.66 | 21.23 |
| 23.86 | 23.82 | 20.30 | 21.52 | 21.52 |
| 21.91 | 23.14 | 20.02 | 22.20 | 21.20 |
| 22.37 | 22.19 | 22.34 | 23.36 | 23.44 |

Assume the population is approximately normal. Provide a point estimate and a 95% confidence interval for the mean number of programming minutes during a half-hour television sitcom.

**OR**

**A1b** On Friday, Wall Street traders were anxiously awaiting the federal government's release of numbers on the January increase in nonfarm payrolls. The early consensus estimate among economists was for a growth of 250,000 new jobs (CNBC, February 3, 2006). However, a sample of 20 economists taken Thursday afternoon provided a sample mean of 266,000 with a sample standard deviation of 24,000. Financial analysts often call such a sample mean, based on late-breaking news, the whisper number. Treat the "consensus estimate" as the population mean. Conduct a hypothesis test to determine whether the whisper number justifies a conclusion of a statistically significant increase in the consensus estimate of economists. Use  $\alpha = .01$  as the level of significance.

**(CILO 2)**

**A2a** The Census Bureau's Current Population Survey shows 28% of individuals, ages 25 and older, have completed four years of college (The New York Times Almanac, 2006). For a sample of 15 individuals, ages 25 and older, answer the following questions:

- a) What is the probability four will have completed four years of college?
- b) What is the probability three or more will have completed four years of college?

OR

A2b The time needed to complete a final examination in a particular college course is normally distributed with a mean of 80 minutes and a standard deviation of 10 minutes. Answer the following questions.

- What is the probability of completing the exam in one hour or less?
- What is the probability that a student will complete the exam in more than 60 minutes but less than 75 minutes?
- Assume that the class has 60 students and that the examination period is 90 minutes in length. How many students do you expect will be unable to complete the exam in the allotted time?

(CILO 3)

A3a NRF/BIG research provided results of a consumer holiday spending survey (USA Today, December 20, 2005). The following data provide the dollar amount of holiday spending for a sample of 25 consumers.

|      |      |      |      |     |
|------|------|------|------|-----|
| 1200 | 850  | 740  | 590  | 340 |
| 450  | 890  | 260  | 610  | 350 |
| 1780 | 180  | 850  | 2050 | 770 |
| 800  | 1090 | 510  | 520  | 220 |
| 1450 | 280  | 1120 | 200  | 350 |

- What is the lowest holiday spending? The highest?
- Use a class width of \$250 to prepare a frequency distribution and a percent frequency distribution for the data.
- Prepare a histogram and comment on the shape of the distribution.
- What observations can you make about holiday spending?

OR

A3b An important application of regression analysis in accounting is in the estimation of cost.

By collecting data on volume and cost and using the least squares method to develop an estimated regression equation relating volume and cost, an accountant can estimate the cost associated with a particular manufacturing volume. Consider the following sample of production volumes and total cost data for a manufacturing operation.

| Production Volume<br>(units) | Total Cost<br>(\$) |
|------------------------------|--------------------|
| 400                          | 4000               |
| 450                          | 5000               |
| 550                          | 5400               |
| 600                          | 5900               |
| 700                          | 6400               |
| 750                          | 7000               |

vol = 3450

cost = 33700

vol<sup>2</sup> = 2077500

cost<sup>2</sup> = 194930000

$$\square \text{ vol} * \text{ cost} = 20090000$$

- a) Use these data to develop an estimated regression equation that could be used to predict the total cost for a given production volume.
- b) What is the variable cost (i.e. cost linked to production volume) per unit produced?
- c) Compute the coefficient of determination. What percentage of the variation in total cost can be explained by production volume?
- d) The company's production schedule shows 500 units must be produced next month. What is the estimated total cost for this operation?

### SECTION B

#### CASE STUDY (5 marks \* 3 questions = 15 Marks)

(CILO 2 and 3)

The Internet has revolutionised business. It is now a common, often preferred, medium for communication and transactions between businesses (B2B), businesses and consumers (B2C), consumers and businesses (C2B), and others.

The Internet has been a significant driver of the rapid globalization of business over the past decade. As a result of this development, Internet-based companies such as Amazon.com and eBay have become household names. Interesting and pertinent questions are, 'How do marketing managers perceive the capabilities of the Internet?' and, of course, 'How do marketing managers use the Internet to improve their marketing performance?'

A study by Elaine Leong, Michael Ewing and Leyland Pitt offers unique insights into these issues and, in particular, the differences before and after the 'dotcom crash'. The survey was conducted over two timed periods spaced exactly one year on either side of the dotcom crash, that is, in April 1999 and April 2001.

The aim of this study was to assess managers' perceptions of the impact of internet marketing on the key elements of marketing strategy: segmentation, targeting, marketing mix (the 'four p's', product, price, place, and promotion) and customer service. Responses were sought on a series of statements about the impact of the Internet on marketing activities based on five-point scales, where 1 is 'to a very little extent' and 5 is 'to a very great extent'. In the 1999 survey, the respondents were required to indicate their perceptions of the Internet's impact on marketing in the next two years. In the 2001 survey, they were asked to look back to 1999 and indicate their perceptions over the past two years. Again in 2001, the respondents were asked to indicate their perceptions of the Internet's impact on marketing over the next two years. Therefore, the three sets of questions may be regarded as (1) a prediction of what was going to happen, (2) what actually happened, and (3) a 'revised' prediction of what would happen in the future.

**Table 1**

Sample Characteristics by Industry categories

|  | 1999(n=170) |     | 2001(n=181) |    |
|--|-------------|-----|-------------|----|
|  | n           | %   | n           | %  |
| <b>Agriculture, forestry, fishing, mining and construction (AMC)</b> | 24          | 14  | 19          | 11 |
| <b>Manufacturing ( Manuf )</b>                                       | 60          | 35  | 44          | 24 |
| <b>Transport, Communication and utilities (TUC)</b>                  | 16          | 9   | 20          | 11 |
| <b>Wholesale and, retail trade (WR)</b>                              | 41          | 24  | 31          | 17 |
| <b>Finance, insurance, real estate and other service (SERV)</b>      | 28          | 16  | 51          | 28 |
| <b>Other and no response</b>   | 1           | 0.6 | 16          | 9  |

**Table 2**

The following table gives a selection of some of the study's results.

|   | Mean predication 1999-2001 (n=170) | Mean actual 1999-2001 (n=181) | Mean predication 2001-2003 (n=181) |
|---|------------------------------------|-------------------------------|------------------------------------|
| <b>Marketing activates Permit online payment</b>                          | 3.36                               | 1.86                          | 3.08                               |
| <b>Provide an enhanced capability to bring new products to the market</b> | 3.48                               | 2.00                          | 3.15                               |
| <b>Marketing definition Target customers globally</b>                     | 3.59                               | 2.07                          | 2.91                               |
| <b>Add new market segments</b>  | 3.12                               | 1.78                          | 2.85                               |
| <b>Develop new market faster</b>  | 2.56                               | 1.59                          | 2.47                               |
| <b>Price more accurately</b>  | 3.56                               | 2.19                          | 3.61                               |

**Discussion**

1. Table 1 summarizes the characteristics of the 1999 and 2001 samples by broad industry categories. What would be an appropriate statistical test to use to ascertain whether category representation had changed between the two samples? What assumptions would be required to conduct this test? Using the information in Table 1, apply this test to determine whether there are any significant differences in representation between the '1999 and 2001 samples for agriculture, forestry, fishing, mining and construction (AMC) and finance, insurance, real estate and other services (SERV).

2. The almost unlimited possibilities offered by the Internet are expected to change the way the market is defined. To examine this issue further, the study assessed managers' perceptions of the degree to which the definition of the market might change in the next two years. The means and standard deviations for four items selected for perceived changes in market

definition are shown in Table 3. Estimate the difference in means between the first statement ('Expand the size of a targeted market') and the third statement ('Define markets more precisely'). Let  $SD = 1.24$ . What conclusion can you make about the disparities in the managers' perceptions of the Internet's impacts on market definition?

3. The authors also wanted to ascertain whether industries differ in how they perceive the impact of the Internet on marketing. Using the data in Table 4, conduct a test to determine whether the manufacturing sector (Manuf) differs from the services sector (SERV) in terms of how they perceive the impact of the Internet on marketing. Discuss and explain your findings.

Table 3

| Statement  | Mean | Standard deviation |
|--|------|--------------------|
| Expand the size a targeted market                        | 3.19 | 1.31               |
| Target new customers in currently served market segments | 3.17 | 1.17               |
| Define markets more precisely                            | 2.65 | 1.18               |
| Eliminate less profitable customers or market segments   | 2.29 | 1.31               |

Table 4 Mean

| Statement                                     | Manuf | SERV |
|---|-------|------|
| Add new segments to target markets            | 2.55  | 3.31 |
| Improve the ability to the customer informed  | 3.77  | 4.53 |
| Better customisation of products and services | 2.42  | 3.51 |

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\sigma_{\bar{x}} = \left(\frac{\sigma}{\sqrt{n}}\right) \sqrt{\frac{N-n}{N-1}}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$(\bar{X}_1 - \bar{X}_2) \pm Z \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

or

$$\bar{x} - z \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z \frac{\sigma}{\sqrt{n}}$$

$$SST = SSB + SSW$$

$$SST = \sum_{i=1}^k \sum_{j=1}^{n_j} (x_{ij} - \bar{x})^2$$

$$SSB = \sum_{i=1}^k n_i (\bar{x}_i - \bar{x})^2$$

$$MSB = \frac{SSB}{k-1}$$

$$SSW = \sum_{i=1}^k \sum_{j=1}^{n_j} (x_{ij} - \bar{x}_i)^2$$

$$MSW = \frac{SSW}{N-k}$$

$$F = \frac{MSB}{MSW}$$

$$\bar{x} \pm t \frac{s}{\sqrt{n}}$$

or

$$\bar{x} - t \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t \frac{s}{\sqrt{n}}$$

$$df = n - 1$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$SST = \sum_{j=1}^c \sum_{i=1}^{n_j} (X_{ij} - \bar{X})^2$$

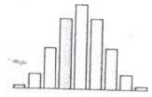
$$SSA = \sum_{j=1}^c n_j (\bar{X}_j - \bar{X})^2$$

$$SSW = \sum_{j=1}^c \sum_{i=1}^{n_j} (X_{ij} - \bar{X}_j)^2$$

# APPENDIX

# A

## EXACT BINOMIAL PROBABILITIES



|     |     | $\pi$ |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
|-----|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $n$ | $X$ | .01   | .02   | .05   | .10   | .15   | .20   | .30   | .40   | .50   | .60   | .70   | .80   | .85   | .90   | .95   | .98   | .99   |
| 2   | 0   | .9801 | .9604 | .9025 | .8100 | .7225 | .6400 | .4900 | .3600 | .2500 | .1600 | .0900 | .0400 | .0225 | .0100 | .0025 | .0004 | .0001 |
|     | 1   | .0198 | .0392 | .0950 | .1800 | .2550 | .3200 | .4200 | .4800 | .5000 | .4800 | .4200 | .3200 | .2550 | .1800 | .0950 | .0392 | .0198 |
|     | 2   | .0001 | .0004 | .0025 | .0100 | .0225 | .0400 | .0900 | .1600 | .2500 | .3600 | .4900 | .6400 | .7225 | .8100 | .9025 | .9604 | .9801 |
| 3   | 0   | .9703 | .9412 | .8574 | .7290 | .6141 | .5120 | .3430 | .2160 | .1250 | .0640 | .0270 | .0080 | .0034 | .0010 | .0001 | —     | —     |
|     | 1   | .0294 | .0576 | .1354 | .2430 | .3251 | .3840 | .4410 | .4320 | .3750 | .2880 | .1890 | .0960 | .0574 | .0270 | .0071 | .0012 | .0003 |
|     | 2   | .0003 | .0012 | .0071 | .0270 | .0574 | .0960 | .1890 | .2880 | .3750 | .4320 | .4410 | .3840 | .3251 | .2430 | .1354 | .0576 | .0294 |
|     | 3   | —     | —     | .0001 | .0010 | .0034 | .0080 | .0270 | .0640 | .1250 | .2160 | .3430 | .5120 | .6141 | .7290 | .8574 | .9412 | .9703 |
| 4   | 0   | .9606 | .9224 | .8145 | .6561 | .5220 | .4096 | .2401 | .1296 | .0625 | .0256 | .0081 | .0016 | .0005 | .0001 | —     | —     | —     |
|     | 1   | .0388 | .0753 | .1715 | .2916 | .3685 | .4096 | .4116 | .3456 | .2500 | .1536 | .0756 | .0256 | .0115 | .0036 | .0005 | —     | —     |
|     | 2   | .0006 | .0023 | .0135 | .0486 | .0975 | .1536 | .2646 | .3456 | .3750 | .3456 | .2646 | .1536 | .0975 | .0486 | .0135 | .0023 | .0006 |
|     | 3   | —     | —     | .0005 | .0036 | .0115 | .0256 | .0756 | .1536 | .2500 | .3456 | .4116 | .4096 | .3685 | .2916 | .1715 | .0753 | .0388 |
|     | 4   | —     | —     | —     | .0001 | .0005 | .0016 | .0081 | .0256 | .0625 | .1296 | .2401 | .4096 | .5220 | .6561 | .8145 | .9224 | .9606 |
| 5   | 0   | .9510 | .9039 | .7738 | .5905 | .4437 | .3277 | .1681 | .0778 | .0313 | .0102 | .0024 | .0003 | .0001 | —     | —     | —     | —     |
|     | 1   | .0480 | .0922 | .2036 | .3281 | .3915 | .4096 | .3602 | .2592 | .1563 | .0768 | .0284 | .0064 | .0022 | .0005 | —     | —     | —     |
|     | 2   | .0010 | .0038 | .0214 | .0729 | .1382 | .2048 | .3087 | .3456 | .3125 | .2304 | .1323 | .0512 | .0244 | .0081 | .0011 | .0001 | —     |
|     | 3   | —     | .0001 | .0011 | .0081 | .0244 | .0512 | .1323 | .2304 | .3125 | .3456 | .3087 | .2048 | .1382 | .0729 | .0214 | .0038 | .0010 |
|     | 4   | —     | —     | —     | .0005 | .0022 | .0064 | .0284 | .0768 | .1563 | .2592 | .3602 | .4096 | .3915 | .3281 | .2036 | .0922 | .0480 |
|     | 5   | —     | —     | —     | —     | .0001 | .0003 | .0024 | .0102 | .0313 | .0778 | .1681 | .3277 | .4437 | .5905 | .7738 | .9039 | .9510 |
| 6   | 0   | .9415 | .8858 | .7351 | .5314 | .3771 | .2621 | .1176 | .0467 | .0156 | .0041 | .0007 | .0001 | —     | —     | —     | —     | —     |
|     | 1   | .0571 | .1085 | .2321 | .3543 | .3993 | .3932 | .3025 | .1866 | .0938 | .0369 | .0102 | .0015 | .0004 | .0001 | —     | —     | —     |
|     | 2   | .0014 | .0055 | .0305 | .0984 | .1762 | .2458 | .3241 | .3110 | .2344 | .1382 | .0595 | .0154 | .0055 | .0012 | .0001 | —     | —     |
|     | 3   | —     | .0002 | .0021 | .0146 | .0415 | .0819 | .1852 | .2765 | .3125 | .2765 | .1852 | .0819 | .0415 | .0146 | .0021 | .0002 | —     |
|     | 4   | —     | —     | .0001 | .0012 | .0055 | .0154 | .0595 | .1382 | .2344 | .3110 | .3241 | .2458 | .1762 | .0984 | .0305 | .0055 | .0014 |
|     | 5   | —     | —     | —     | .0001 | .0004 | .0015 | .0102 | .0369 | .0938 | .1866 | .3025 | .3932 | .3993 | .3543 | .2321 | .1085 | .0571 |
|     | 6   | —     | —     | —     | —     | .0001 | .0007 | .0041 | .0156 | .0467 | .1176 | .2621 | .3771 | .5314 | .7351 | .8858 | .9415 | —     |
| 7   | 0   | .9321 | .8681 | .6983 | .4783 | .3206 | .2097 | .0824 | .0280 | .0078 | .0016 | .0002 | —     | —     | —     | —     | —     | —     |
|     | 1   | .0659 | .1240 | .2573 | .3720 | .3960 | .3670 | .2471 | .1306 | .0547 | .0172 | .0036 | .0004 | .0001 | —     | —     | —     | —     |
|     | 2   | .0020 | .0076 | .0406 | .1240 | .2097 | .2753 | .3177 | .2613 | .1641 | .0774 | .0250 | .0043 | .0012 | .0002 | —     | —     | —     |
|     | 3   | —     | .0003 | .0036 | .0230 | .0617 | .1147 | .2269 | .2903 | .2734 | .1935 | .0972 | .0287 | .0109 | .0026 | .0002 | —     | —     |
|     | 4   | —     | —     | .0002 | .0026 | .0109 | .0287 | .0972 | .1935 | .2734 | .2903 | .2269 | .1147 | .0617 | .0230 | .0036 | .0003 | —     |
|     | 5   | —     | —     | —     | .0002 | .0012 | .0043 | .0250 | .0774 | .1641 | .2613 | .3177 | .2753 | .2097 | .1240 | .0406 | .0076 | .0020 |
|     | 6   | —     | —     | —     | .0001 | .0004 | .0036 | .0172 | .0547 | .1306 | .2471 | .3670 | .3960 | .3720 | .2573 | .1240 | .0659 | —     |
|     | 7   | —     | —     | —     | —     | —     | .0002 | .0016 | .0078 | .0280 | .0824 | .2097 | .3206 | .4783 | .6983 | .8681 | .9321 | —     |
| 8   | 0   | .9227 | .8508 | .6634 | .4305 | .2725 | .1678 | .0576 | .0168 | .0039 | .0007 | .0001 | —     | —     | —     | —     | —     | —     |
|     | 1   | .0746 | .1389 | .2793 | .3826 | .3847 | .3355 | .1977 | .0896 | .0313 | .0079 | .0012 | .0001 | —     | —     | —     | —     | —     |
|     | 2   | .0026 | .0099 | .0515 | .1488 | .2376 | .2936 | .2965 | .2090 | .1094 | .0413 | .0100 | .0011 | .0002 | —     | —     | —     | —     |
|     | 3   | .0001 | .0004 | .0054 | .0331 | .0839 | .1468 | .2541 | .2787 | .2188 | .1239 | .0467 | .0092 | .0026 | .0004 | —     | —     | —     |
|     | 4   | —     | —     | .0004 | .0046 | .0185 | .0459 | .1361 | .2322 | .2734 | .2322 | .1361 | .0459 | .0185 | .0046 | .0004 | —     | —     |
|     | 5   | —     | —     | —     | .0004 | .0026 | .0092 | .0467 | .1239 | .2188 | .2787 | .2541 | .1468 | .0839 | .0331 | .0054 | .0004 | .0001 |
|     | 6   | —     | —     | —     | —     | .0002 | .0011 | .0100 | .0413 | .1094 | .2090 | .2965 | .2936 | .2376 | .1488 | .0515 | .0099 | .0026 |
|     | 7   | —     | —     | —     | —     | —     | .0001 | .0012 | .0079 | .0313 | .0896 | .1977 | .3355 | .3847 | .3826 | .2793 | .1389 | .0746 |
|     | 8   | —     | —     | —     | —     | —     | —     | .0001 | .0007 | .0039 | .0168 | .0576 | .1678 | .2725 | .4305 | .6634 | .8508 | .9227 |
| 9   | 0   | .9135 | .8337 | .6302 | .3874 | .2316 | .1342 | .0404 | .0101 | .0020 | .0003 | —     | —     | —     | —     | —     | —     | —     |
|     | 1   | .0830 | .1531 | .2985 | .3874 | .3679 | .3020 | .1556 | .0605 | .0176 | .0035 | .0004 | —     | —     | —     | —     | —     | —     |
|     | 2   | .0034 | .0125 | .0629 | .1722 | .2597 | .3020 | .2668 | .1612 | .0703 | .0212 | .0039 | .0003 | —     | —     | —     | —     | —     |
|     | 3   | .0001 | .0006 | .0077 | .0446 | .1069 | .1762 | .2668 | .2508 | .1641 | .0743 | .0210 | .0028 | .0006 | .0001 | —     | —     | —     |
|     | 4   | —     | —     | .0006 | .0074 | .0283 | .0661 | .1715 | .2508 | .2461 | .1672 | .0735 | .0165 | .0050 | .0008 | —     | —     | —     |
|     | 5   | —     | —     | —     | .0008 | .0050 | .0165 | .0735 | .1672 | .2461 | .2508 | .1715 | .0661 | .0283 | .0074 | .0006 | —     | —     |
|     | 6   | —     | —     | —     | .0001 | .0006 | .0028 | .0210 | .0743 | .1641 | .2508 | .2668 | .1762 | .1069 | .0446 | .0077 | .0006 | .0001 |
|     | 7   | —     | —     | —     | —     | —     | .0003 | .0039 | .0212 | .0703 | .1612 | .2668 | .3020 | .2597 | .1722 | .0629 | .0125 | .0034 |
|     | 8   | —     | —     | —     | —     | —     | —     | .0004 | .0035 | .0176 | .0605 | .1556 | .3020 | .3679 | .3874 | .2985 | .1531 | .0830 |
|     | 9   | —     | —     | —     | —     | —     | —     | —     | .0003 | .0020 | .0101 | .0404 | .1342 | .2316 | .3874 | .6302 | .8337 | .9135 |

$\pi$

| $n$ | $X$ | .01   | .02   | .05   | .10   | .15   | .20   | .30   | .40   | .50   | .60   | .70   | .80   | .85   | .90   | .95   | .98   | .99   |
|-----|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10  | 0   | .9044 | .8171 | .5987 | .3487 | .1969 | .1074 | .0282 | .0060 | .0010 | .0001 | —     | —     | —     | —     | —     | —     | —     |
| 10  | 1   | .0914 | .1667 | .3151 | .3874 | .3474 | .2684 | .1211 | .0403 | .0098 | .0016 | .0001 | —     | —     | —     | —     | —     | —     |
| 10  | 2   | .0042 | .0153 | .0746 | .1937 | .2759 | .3020 | .2335 | .1209 | .0439 | .0106 | .0014 | .0001 | —     | —     | —     | —     | —     |
| 10  | 3   | .0001 | .0008 | .0105 | .0574 | .1298 | .2013 | .2668 | .2150 | .1172 | .0425 | .0090 | .0008 | .0001 | —     | —     | —     | —     |
| 10  | 4   | —     | —     | .0010 | .0112 | .0401 | .0881 | .2001 | .2508 | .2051 | .1115 | .0368 | .0055 | .0012 | .0001 | —     | —     | —     |
| 10  | 5   | —     | —     | .0001 | .0015 | .0085 | .0264 | .1029 | .2007 | .2461 | .2007 | .1029 | .0264 | .0085 | .0015 | .0001 | —     | —     |
| 10  | 6   | —     | —     | —     | .0001 | .0012 | .0055 | .0368 | .1115 | .2051 | .2508 | .2001 | .0881 | .0401 | .0112 | .0010 | —     | —     |
| 10  | 7   | —     | —     | —     | —     | .0001 | .0008 | .0090 | .0425 | .1172 | .2150 | .2668 | .2013 | .1298 | .0574 | .0105 | .0008 | .0001 |
| 10  | 8   | —     | —     | —     | —     | —     | .0001 | .0014 | .0106 | .0439 | .1209 | .2335 | .3020 | .2759 | .1937 | .0746 | .0153 | .0042 |
| 10  | 9   | —     | —     | —     | —     | —     | —     | .0001 | .0016 | .0098 | .0403 | .1211 | .2684 | .3474 | .3874 | .3151 | .1667 | .0914 |
| 10  | 10  | —     | —     | —     | —     | —     | —     | —     | .0001 | .0010 | .0060 | .0282 | .1074 | .1969 | .3487 | .5987 | .8171 | .9044 |
| 12  | 0   | .8864 | .7847 | .5404 | .2824 | .1422 | .0687 | .0138 | .0022 | .0002 | —     | —     | —     | —     | —     | —     | —     | —     |
| 12  | 1   | .1074 | .1922 | .3413 | .3766 | .3012 | .2062 | .0712 | .0174 | .0029 | .0003 | —     | —     | —     | —     | —     | —     | —     |
| 12  | 2   | .0060 | .0216 | .0988 | .2301 | .2924 | .2835 | .1678 | .0639 | .0161 | .0025 | .0002 | —     | —     | —     | —     | —     | —     |
| 12  | 3   | .0002 | .0015 | .0173 | .0852 | .1720 | .2362 | .2397 | .1419 | .0537 | .0125 | .0015 | .0001 | —     | —     | —     | —     | —     |
| 12  | 4   | —     | .0001 | .0021 | .0213 | .0683 | .1329 | .2311 | .2128 | .1208 | .0420 | .0078 | .0005 | .0001 | —     | —     | —     | —     |
| 12  | 5   | —     | —     | .0002 | .0038 | .0193 | .0532 | .1585 | .2270 | .1934 | .1009 | .0291 | .0033 | .0006 | —     | —     | —     | —     |
| 12  | 6   | —     | —     | —     | .0005 | .0040 | .0155 | .0792 | .1766 | .2256 | .1766 | .0792 | .0155 | .0040 | .0005 | —     | —     | —     |
| 12  | 7   | —     | —     | —     | —     | .0006 | .0033 | .0291 | .1009 | .1934 | .2270 | .1585 | .0532 | .0193 | .0038 | .0002 | —     | —     |
| 12  | 8   | —     | —     | —     | —     | .0001 | .0005 | .0078 | .0420 | .1208 | .2128 | .2311 | .1329 | .0683 | .0213 | .0021 | .0001 | —     |
| 12  | 9   | —     | —     | —     | —     | —     | .0001 | .0015 | .0125 | .0537 | .1419 | .2397 | .2362 | .1720 | .0852 | .0173 | .0015 | .0002 |
| 12  | 10  | —     | —     | —     | —     | —     | —     | .0002 | .0025 | .0161 | .0639 | .1678 | .2835 | .2924 | .2301 | .0988 | .0216 | .0060 |
| 12  | 11  | —     | —     | —     | —     | —     | —     | —     | .0003 | .0029 | .0174 | .0712 | .2062 | .3012 | .3766 | .3413 | .1922 | .1074 |
| 12  | 12  | —     | —     | —     | —     | —     | —     | —     | —     | .0002 | .0022 | .0138 | .0687 | .1422 | .2824 | .5404 | .7847 | .8864 |
| 14  | 0   | .8687 | .7536 | .4877 | .2288 | .1028 | .0440 | .0068 | .0008 | .0001 | —     | —     | —     | —     | —     | —     | —     | —     |
| 14  | 1   | .1229 | .2153 | .3593 | .3559 | .2539 | .1539 | .0407 | .0073 | .0009 | .0001 | —     | —     | —     | —     | —     | —     | —     |
| 14  | 2   | .0081 | .0286 | .1229 | .2570 | .2912 | .2501 | .1134 | .0317 | .0056 | .0005 | —     | —     | —     | —     | —     | —     | —     |
| 14  | 3   | .0003 | .0023 | .0259 | .1142 | .2056 | .2501 | .1943 | .0845 | .0222 | .0033 | .0002 | —     | —     | —     | —     | —     | —     |
| 14  | 4   | —     | .0001 | .0037 | .0349 | .0998 | .1720 | .2290 | .1549 | .0611 | .0136 | .0014 | —     | —     | —     | —     | —     | —     |
| 14  | 5   | —     | —     | .0004 | .0078 | .0352 | .0860 | .1963 | .2066 | .1222 | .0408 | .0066 | .0003 | —     | —     | —     | —     | —     |
| 14  | 6   | —     | —     | —     | .0013 | .0093 | .0322 | .1262 | .2066 | .1833 | .0918 | .0232 | .0020 | .0003 | —     | —     | —     | —     |
| 14  | 7   | —     | —     | —     | .0002 | .0019 | .0092 | .0618 | .1574 | .2095 | .1574 | .0618 | .0092 | .0019 | .0002 | —     | —     | —     |
| 14  | 8   | —     | —     | —     | —     | .0003 | .0020 | .0232 | .0918 | .1833 | .2066 | .1262 | .0322 | .0093 | .0013 | —     | —     | —     |
| 14  | 9   | —     | —     | —     | —     | —     | .0003 | .0066 | .0408 | .1222 | .2066 | .1963 | .0860 | .0352 | .0078 | .0004 | —     | —     |
| 14  | 10  | —     | —     | —     | —     | —     | —     | .0014 | .0136 | .0611 | .1549 | .2290 | .1720 | .0998 | .0349 | .0037 | .0001 | —     |
| 14  | 11  | —     | —     | —     | —     | —     | —     | .0002 | .0033 | .0222 | .0845 | .1943 | .2501 | .2056 | .1142 | .0259 | .0023 | .0003 |
| 14  | 12  | —     | —     | —     | —     | —     | —     | —     | .0005 | .0056 | .0317 | .1134 | .2501 | .2912 | .2570 | .1229 | .0286 | .0081 |
| 14  | 13  | —     | —     | —     | —     | —     | —     | —     | .0001 | .0009 | .0073 | .0407 | .1539 | .2539 | .3559 | .3593 | .2153 | .1229 |
| 14  | 14  | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0008 | .0068 | .0440 | .1028 | .2288 | .4877 | .7536 | .8687 |
| 16  | 0   | .8515 | .7238 | .4401 | .1853 | .0743 | .0281 | .0033 | .0003 | —     | —     | —     | —     | —     | —     | —     | —     | —     |
| 16  | 1   | .1376 | .2363 | .3706 | .3294 | .2097 | .1126 | .0228 | .0030 | .0002 | —     | —     | —     | —     | —     | —     | —     | —     |
| 16  | 2   | .0104 | .0362 | .1463 | .2745 | .2775 | .2111 | .0732 | .0150 | .0018 | .0001 | —     | —     | —     | —     | —     | —     | —     |
| 16  | 3   | .0005 | .0034 | .0359 | .1423 | .2285 | .2463 | .1465 | .0468 | .0085 | .0008 | —     | —     | —     | —     | —     | —     | —     |
| 16  | 4   | —     | .0002 | .0061 | .0514 | .1311 | .2001 | .2040 | .1014 | .0278 | .0040 | .0002 | —     | —     | —     | —     | —     | —     |
| 16  | 5   | —     | —     | .0008 | .0137 | .0555 | .1201 | .2099 | .1623 | .0667 | .0142 | .0013 | —     | —     | —     | —     | —     | —     |
| 16  | 6   | —     | —     | .0001 | .0028 | .0180 | .0550 | .1649 | .1983 | .1222 | .0392 | .0056 | .0002 | —     | —     | —     | —     | —     |
| 16  | 7   | —     | —     | —     | .0004 | .0045 | .0197 | .1010 | .1889 | .1746 | .0840 | .0185 | .0012 | .0001 | —     | —     | —     | —     |
| 16  | 8   | —     | —     | —     | .0001 | .0009 | .0055 | .0487 | .1417 | .1964 | .1417 | .0487 | .0055 | .0009 | .0001 | —     | —     | —     |
| 16  | 9   | —     | —     | —     | —     | .0001 | .0012 | .0185 | .0840 | .1746 | .1889 | .1010 | .0197 | .0045 | .0004 | —     | —     | —     |
| 16  | 10  | —     | —     | —     | —     | —     | .0002 | .0056 | .0392 | .1222 | .1983 | .1649 | .0550 | .0180 | .0028 | .0001 | —     | —     |
| 16  | 11  | —     | —     | —     | —     | —     | —     | .0013 | .0142 | .0667 | .1623 | .2099 | .1201 | .0555 | .0137 | .0008 | —     | —     |
| 16  | 12  | —     | —     | —     | —     | —     | —     | .0002 | .0040 | .0278 | .1014 | .2040 | .2001 | .1311 | .0514 | .0061 | .0002 | —     |
| 16  | 13  | —     | —     | —     | —     | —     | —     | —     | .0008 | .0085 | .0468 | .1465 | .2463 | .2285 | .1423 | .0359 | .0034 | .0005 |
| 16  | 14  | —     | —     | —     | —     | —     | —     | —     | .0001 | .0018 | .0150 | .0732 | .2111 | .2775 | .2745 | .1463 | .0362 | .0104 |
| 16  | 15  | —     | —     | —     | —     | —     | —     | —     | —     | .0002 | .0030 | .0228 | .1126 | .2097 | .3294 | .3706 | .2363 | .1376 |
| 16  | 16  | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0003 | .0033 | .0281 | .0743 | .1853 | .4401 | .7238 | .8515 |

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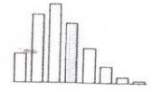
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# APPENDIX

# B

## EXACT POISSON PROBABILITIES



| X | $\lambda$ |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
|---|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|   | 0.1       | 0.2   | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   | 0.8   | 0.9   | 1.0   | 1.1   | 1.2   | 1.3   | 1.4   | 1.5   |
| 0 | .9048     | .8187 | .7408 | .6703 | .6065 | .5488 | .4966 | .4493 | .4066 | .3679 | .3329 | .3012 | .2725 | .2466 | .2231 |
| 1 | .0905     | .1637 | .2222 | .2681 | .3033 | .3293 | .3476 | .3595 | .3659 | .3679 | .3662 | .3614 | .3543 | .3452 | .3347 |
| 2 | .0045     | .0164 | .0333 | .0536 | .0758 | .0988 | .1217 | .1438 | .1647 | .1839 | .2014 | .2169 | .2303 | .2417 | .2510 |
| 3 | .0002     | .0011 | .0033 | .0072 | .0126 | .0198 | .0284 | .0383 | .0494 | .0613 | .0738 | .0867 | .0998 | .1128 | .1255 |
| 4 | —         | .0001 | .0003 | .0007 | .0016 | .0030 | .0050 | .0077 | .0111 | .0153 | .0203 | .0260 | .0324 | .0395 | .0471 |
| 5 | —         | —     | —     | .0001 | .0002 | .0004 | .0007 | .0012 | .0020 | .0031 | .0045 | .0062 | .0084 | .0111 | .0141 |
| 6 | —         | —     | —     | —     | —     | —     | .0001 | .0002 | .0003 | .0005 | .0008 | .0012 | .0018 | .0026 | .0035 |
| 7 | —         | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0002 | .0003 | .0005 | .0008 |
| 8 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0001 |

| X  | $\lambda$ |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
|----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|    | 1.6       | 1.7   | 1.8   | 1.9   | 2.0   | 2.1   | 2.2   | 2.3   | 2.4   | 2.5   | 2.6   | 2.7   | 2.8   | 2.9   | 3.0   |
| 0  | .2019     | .1827 | .1653 | .1496 | .1353 | .1225 | .1108 | .1003 | .0907 | .0821 | .0743 | .0672 | .0608 | .0550 | .0498 |
| 1  | .3230     | .3106 | .2975 | .2842 | .2707 | .2572 | .2438 | .2306 | .2177 | .2052 | .1931 | .1815 | .1703 | .1596 | .1494 |
| 2  | .2584     | .2640 | .2678 | .2700 | .2707 | .2700 | .2681 | .2652 | .2613 | .2565 | .2510 | .2450 | .2384 | .2314 | .2240 |
| 3  | .1378     | .1496 | .1607 | .1710 | .1804 | .1890 | .1966 | .2033 | .2090 | .2138 | .2176 | .2205 | .2225 | .2237 | .2240 |
| 4  | .0551     | .0636 | .0723 | .0812 | .0902 | .0992 | .1082 | .1169 | .1254 | .1336 | .1414 | .1488 | .1557 | .1622 | .1680 |
| 5  | .0176     | .0216 | .0260 | .0309 | .0361 | .0417 | .0476 | .0538 | .0602 | .0668 | .0735 | .0804 | .0872 | .0940 | .1008 |
| 6  | .0047     | .0061 | .0078 | .0098 | .0120 | .0146 | .0174 | .0206 | .0241 | .0278 | .0319 | .0362 | .0407 | .0455 | .0504 |
| 7  | .0011     | .0015 | .0020 | .0027 | .0034 | .0044 | .0055 | .0068 | .0083 | .0099 | .0118 | .0139 | .0163 | .0188 | .0216 |
| 8  | .0002     | .0003 | .0005 | .0006 | .0009 | .0011 | .0015 | .0019 | .0025 | .0031 | .0038 | .0047 | .0057 | .0068 | .0081 |
| 9  | —         | .0001 | .0001 | .0001 | .0002 | .0003 | .0004 | .0005 | .0007 | .0009 | .0011 | .0014 | .0018 | .0022 | .0027 |
| 10 | —         | —     | —     | —     | —     | .0001 | .0001 | .0001 | .0002 | .0002 | .0003 | .0004 | .0005 | .0006 | .0008 |
| 11 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0001 | .0002 | .0002 |
| 12 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 |

| X | $\lambda$ |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
|---|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|   | 3.1       | 3.2   | 3.3   | 3.4   | 3.5   | 3.6   | 3.7   | 3.8   | 3.9   | 4.0   | 4.1   | 4.2   | 4.3   | 4.4   | 4.5   |
| 0 | .0450     | .0408 | .0369 | .0334 | .0302 | .0273 | .0247 | .0224 | .0202 | .0183 | .0166 | .0150 | .0136 | .0123 | .0111 |
| 1 | .3797     | .3304 | .2817 | .2335 | .1857 | .1384 | .0915 | .0450 | .0089 | .0733 | .0679 | .0630 | .0583 | .0540 | .0500 |
| 2 | .2165     | .2087 | .2008 | .1929 | .1850 | .1771 | .1692 | .1615 | .1539 | .1465 | .1393 | .1323 | .1254 | .1188 | .1125 |
| 3 | .2237     | .2226 | .2209 | .2186 | .2158 | .2125 | .2087 | .2046 | .2001 | .1954 | .1904 | .1852 | .1798 | .1743 | .1687 |
| 4 | .1733     | .1781 | .1823 | .1858 | .1888 | .1912 | .1931 | .1944 | .1951 | .1954 | .1951 | .1944 | .1933 | .1917 | .1898 |
| 5 | .1075     | .1140 | .1203 | .1264 | .1322 | .1377 | .1429 | .1477 | .1522 | .1563 | .1600 | .1633 | .1662 | .1687 | .1708 |
| 6 | .0555     | .0608 | .0662 | .0716 | .0771 | .0826 | .0881 | .0936 | .0989 | .1042 | .1093 | .1143 | .1191 | .1237 | .1281 |
| 7 | .0246     | .0278 | .0312 | .0348 | .0385 | .0425 | .0466 | .0508 | .0551 | .0595 | .0640 | .0686 | .0732 | .0778 | .0824 |
| 8 | .0095     | .0111 | .0129 | .0148 | .0169 | .0191 | .0215 | .0241 | .0269 | .0298 | .0328 | .0360 | .0393 | .0428 | .0463 |
| 9 | .0033     | .0040 | .0047 | .0056 | .0066 | .0076 | .0089 | .0102 | .0116 | .0132 | .0150 | .0168 | .0188 | .0209 | .0232 |
| 0 | .0010     | .0013 | .0016 | .0019 | .0023 | .0028 | .0033 | .0039 | .0045 | .0053 | .0061 | .0071 | .0081 | .0092 | .0104 |
| 1 | .0003     | .0004 | .0005 | .0006 | .0007 | .0009 | .0011 | .0013 | .0016 | .0019 | .0023 | .0027 | .0032 | .0037 | .0043 |
| 2 | .0001     | .0001 | .0001 | .0002 | .0002 | .0003 | .0003 | .0004 | .0005 | .0006 | .0008 | .0009 | .0011 | .0013 | .0016 |
| 3 | —         | —     | —     | —     | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 | .0003 | .0004 | .0005 | .0006 |
| 4 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0001 | .0001 | .0002 |
| 5 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 |

$\lambda$

| X  | 4.6   | 4.7   | 4.8   | 4.9   | 5.0   | 5.1   | 5.2   | 5.3   | 5.4   | 5.5   | 5.6   | 5.7   | 5.8   | 5.9   | 6.0   |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0  | .0101 | .0091 | .0082 | .0074 | .0067 | .0061 | .0055 | .0050 | .0045 | .0041 | .0037 | .0033 | .0030 | .0027 | .0025 |
| 1  | .0462 | .0427 | .0395 | .0365 | .0337 | .0311 | .0287 | .0265 | .0244 | .0225 | .0207 | .0191 | .0176 | .0162 | .0149 |
| 2  | .1063 | .1005 | .0948 | .0894 | .0842 | .0793 | .0746 | .0701 | .0659 | .0618 | .0580 | .0544 | .0509 | .0477 | .0446 |
| 3  | .1631 | .1574 | .1517 | .1460 | .1404 | .1348 | .1293 | .1239 | .1185 | .1133 | .1082 | .1033 | .0985 | .0938 | .0892 |
| 4  | .1875 | .1849 | .1820 | .1789 | .1755 | .1719 | .1681 | .1641 | .1600 | .1558 | .1515 | .1472 | .1428 | .1383 | .1339 |
| 5  | .1725 | .1738 | .1747 | .1753 | .1755 | .1753 | .1748 | .1740 | .1728 | .1714 | .1697 | .1678 | .1656 | .1632 | .1606 |
| 6  | .1323 | .1362 | .1398 | .1432 | .1462 | .1490 | .1515 | .1537 | .1555 | .1571 | .1584 | .1594 | .1601 | .1605 | .1606 |
| 7  | .0869 | .0914 | .0959 | .1002 | .1044 | .1086 | .1125 | .1163 | .1200 | .1234 | .1267 | .1298 | .1326 | .1353 | .1377 |
| 8  | .0500 | .0537 | .0575 | .0614 | .0653 | .0692 | .0731 | .0771 | .0810 | .0849 | .0887 | .0925 | .0962 | .0998 | .1033 |
| 9  | .0255 | .0281 | .0307 | .0334 | .0363 | .0392 | .0423 | .0454 | .0486 | .0519 | .0552 | .0586 | .0620 | .0654 | .0688 |
| 10 | .0118 | .0132 | .0147 | .0164 | .0181 | .0200 | .0220 | .0241 | .0262 | .0285 | .0309 | .0334 | .0359 | .0386 | .0413 |
| 11 | .0049 | .0056 | .0064 | .0073 | .0082 | .0093 | .0104 | .0116 | .0129 | .0143 | .0157 | .0173 | .0190 | .0207 | .0225 |
| 12 | .0019 | .0022 | .0026 | .0030 | .0034 | .0039 | .0045 | .0051 | .0058 | .0065 | .0073 | .0082 | .0092 | .0102 | .0113 |
| 13 | .0007 | .0008 | .0009 | .0011 | .0013 | .0015 | .0018 | .0021 | .0024 | .0028 | .0032 | .0036 | .0041 | .0046 | .0052 |
| 14 | .0002 | .0003 | .0003 | .0004 | .0005 | .0006 | .0007 | .0008 | .0009 | .0011 | .0013 | .0015 | .0017 | .0019 | .0022 |
| 15 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 | .0003 | .0003 | .0004 | .0005 | .0006 | .0007 | .0008 | .0009 |
| 16 | —     | —     | —     | —     | —     | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 | .0003 | .0003 |
| 17 | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0001 | .0001 | .0001 |

$\lambda$

| X  | 6.1   | 6.2   | 6.3   | 6.4   | 6.5   | 6.6   | 6.7   | 6.8   | 6.9   | 7.0   | 7.1   | 7.2   | 7.3   | 7.4   | 7.5   |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0  | .0022 | .0020 | .0018 | .0017 | .0015 | .0014 | .0012 | .0011 | .0010 | .0009 | .0008 | .0007 | .0007 | .0006 | .0006 |
| 1  | .0137 | .0126 | .0116 | .0106 | .0098 | .0090 | .0082 | .0076 | .0070 | .0064 | .0059 | .0054 | .0049 | .0045 | .0041 |
| 2  | .0417 | .0390 | .0364 | .0340 | .0318 | .0296 | .0276 | .0258 | .0240 | .0223 | .0208 | .0194 | .0180 | .0167 | .0156 |
| 3  | .0848 | .0806 | .0765 | .0726 | .0688 | .0652 | .0617 | .0584 | .0552 | .0521 | .0492 | .0464 | .0438 | .0413 | .0389 |
| 4  | .1294 | .1249 | .1205 | .1162 | .1118 | .1076 | .1034 | .0992 | .0952 | .0912 | .0874 | .0836 | .0799 | .0764 | .0729 |
| 5  | .1579 | .1549 | .1519 | .1487 | .1454 | .1420 | .1385 | .1349 | .1314 | .1277 | .1241 | .1204 | .1167 | .1130 | .1094 |
| 6  | .1605 | .1601 | .1595 | .1586 | .1575 | .1562 | .1546 | .1529 | .1511 | .1490 | .1468 | .1445 | .1420 | .1394 | .1367 |
| 7  | .1399 | .1418 | .1435 | .1450 | .1462 | .1472 | .1480 | .1486 | .1489 | .1490 | .1489 | .1486 | .1481 | .1474 | .1465 |
| 8  | .1066 | .1099 | .1130 | .1160 | .1188 | .1215 | .1240 | .1263 | .1284 | .1304 | .1321 | .1337 | .1351 | .1363 | .1373 |
| 9  | .0723 | .0757 | .0791 | .0825 | .0858 | .0891 | .0923 | .0954 | .0985 | .1014 | .1042 | .1070 | .1096 | .1121 | .1144 |
| 10 | .0441 | .0469 | .0498 | .0528 | .0558 | .0588 | .0618 | .0649 | .0679 | .0710 | .0740 | .0770 | .0800 | .0829 | .0858 |
| 11 | .0244 | .0265 | .0285 | .0307 | .0330 | .0353 | .0377 | .0401 | .0426 | .0452 | .0478 | .0504 | .0531 | .0558 | .0585 |
| 12 | .0124 | .0137 | .0150 | .0164 | .0179 | .0194 | .0210 | .0227 | .0245 | .0263 | .0283 | .0303 | .0323 | .0344 | .0366 |
| 13 | .0058 | .0065 | .0073 | .0081 | .0089 | .0099 | .0108 | .0119 | .0130 | .0142 | .0154 | .0168 | .0181 | .0196 | .0211 |
| 14 | .0025 | .0029 | .0033 | .0037 | .0041 | .0046 | .0052 | .0058 | .0064 | .0071 | .0078 | .0086 | .0095 | .0104 | .0113 |
| 15 | .0010 | .0012 | .0014 | .0016 | .0018 | .0020 | .0023 | .0026 | .0029 | .0033 | .0037 | .0041 | .0046 | .0051 | .0057 |
| 16 | .0004 | .0005 | .0005 | .0006 | .0007 | .0008 | .0010 | .0011 | .0013 | .0014 | .0016 | .0019 | .0021 | .0024 | .0026 |
| 17 | .0001 | .0002 | .0002 | .0002 | .0003 | .0003 | .0004 | .0004 | .0005 | .0006 | .0007 | .0008 | .0009 | .0010 | .0012 |
| 18 | —     | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 | .0003 | .0003 | .0004 | .0004 | .0005 |
| 19 | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 |
| 20 | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0001 |

Appendix B (continued)

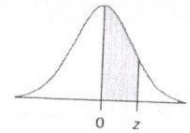
|    | $\lambda$ |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
|----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|    | 8.0       | 8.5   | 9.0   | 9.5   | 10.0  | 11.0  | 12.0  | 13.0  | 14.0  | 15.0  | 16.0  | 17.0  | 18.0  | 19.0  | 20.0  |
| 0  | .0003     | .0002 | .0001 | .0001 | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     |
| 1  | .0027     | .0017 | .0011 | .0007 | .0005 | .0002 | .0001 | —     | —     | —     | —     | —     | —     | —     | —     |
| 2  | .0107     | .0074 | .0050 | .0034 | .0023 | .0010 | .0004 | .0002 | .0001 | —     | —     | —     | —     | —     | —     |
| 3  | .0286     | .0208 | .0150 | .0107 | .0076 | .0037 | .0018 | .0008 | .0004 | .0002 | .0001 | —     | —     | —     | —     |
| 4  | .0573     | .0443 | .0337 | .0254 | .0189 | .0102 | .0053 | .0027 | .0013 | .0006 | .0003 | .0001 | .0001 | —     | —     |
| 5  | .0916     | .0752 | .0607 | .0483 | .0378 | .0224 | .0127 | .0070 | .0037 | .0019 | .0010 | .0005 | .0002 | .0001 | .0001 |
| 6  | .1221     | .1066 | .0911 | .0764 | .0631 | .0411 | .0255 | .0152 | .0087 | .0048 | .0026 | .0014 | .0007 | .0004 | .0002 |
| 7  | .1396     | .1294 | .1171 | .1037 | .0901 | .0646 | .0437 | .0281 | .0174 | .0104 | .0060 | .0034 | .0019 | .0010 | .0005 |
| 8  | .1396     | .1375 | .1318 | .1232 | .1126 | .0888 | .0655 | .0457 | .0304 | .0194 | .0120 | .0072 | .0042 | .0024 | .0013 |
| 9  | .1241     | .1299 | .1318 | .1300 | .1251 | .1085 | .0874 | .0661 | .0473 | .0324 | .0213 | .0135 | .0083 | .0050 | .0029 |
| 10 | .0993     | .1104 | .1186 | .1235 | .1251 | .1194 | .1048 | .0859 | .0663 | .0486 | .0341 | .0230 | .0150 | .0095 | .0058 |
| 11 | .0722     | .0853 | .0970 | .1067 | .1137 | .1194 | .1144 | .1015 | .0844 | .0663 | .0496 | .0355 | .0245 | .0164 | .0106 |
| 12 | .0481     | .0604 | .0728 | .0844 | .0948 | .1094 | .1144 | .1099 | .0984 | .0829 | .0661 | .0504 | .0368 | .0259 | .0176 |
| 13 | .0296     | .0395 | .0504 | .0617 | .0729 | .0926 | .1056 | .1099 | .1060 | .0956 | .0814 | .0658 | .0509 | .0378 | .0271 |
| 14 | .0169     | .0240 | .0324 | .0419 | .0521 | .0728 | .0905 | .1021 | .1060 | .1024 | .0930 | .0800 | .0655 | .0514 | .0387 |
| 15 | .0090     | .0136 | .0194 | .0265 | .0347 | .0534 | .0724 | .0885 | .0989 | .1024 | .0992 | .0906 | .0786 | .0650 | .0516 |
| 16 | .0045     | .0072 | .0109 | .0157 | .0217 | .0367 | .0543 | .0719 | .0866 | .0960 | .0992 | .0963 | .0884 | .0772 | .0646 |
| 17 | .0021     | .0036 | .0058 | .0088 | .0128 | .0237 | .0383 | .0550 | .0713 | .0847 | .0934 | .0963 | .0936 | .0863 | .0760 |
| 18 | .0009     | .0017 | .0029 | .0046 | .0071 | .0145 | .0255 | .0397 | .0554 | .0706 | .0830 | .0909 | .0936 | .0911 | .0844 |
| 19 | .0004     | .0008 | .0014 | .0023 | .0037 | .0084 | .0161 | .0272 | .0409 | .0557 | .0699 | .0814 | .0887 | .0911 | .0888 |
| 20 | .0002     | .0003 | .0006 | .0011 | .0019 | .0046 | .0097 | .0177 | .0286 | .0418 | .0559 | .0692 | .0798 | .0866 | .0888 |
| 21 | .0001     | .0001 | .0003 | .0005 | .0009 | .0024 | .0055 | .0109 | .0191 | .0299 | .0426 | .0560 | .0684 | .0783 | .0846 |
| 22 | —         | .0001 | .0001 | .0002 | .0004 | .0012 | .0030 | .0065 | .0121 | .0204 | .0310 | .0433 | .0560 | .0676 | .0769 |
| 23 | —         | —     | —     | .0001 | .0002 | .0006 | .0016 | .0037 | .0074 | .0133 | .0216 | .0320 | .0438 | .0559 | .0669 |
| 24 | —         | —     | —     | —     | .0001 | .0003 | .0008 | .0020 | .0043 | .0083 | .0144 | .0226 | .0328 | .0442 | .0557 |
| 25 | —         | —     | —     | —     | —     | .0001 | .0004 | .0010 | .0024 | .0050 | .0092 | .0154 | .0237 | .0336 | .0446 |
| 26 | —         | —     | —     | —     | —     | —     | .0002 | .0005 | .0013 | .0029 | .0057 | .0101 | .0164 | .0246 | .0343 |
| 27 | —         | —     | —     | —     | —     | —     | .0001 | .0002 | .0007 | .0016 | .0034 | .0063 | .0109 | .0173 | .0254 |
| 28 | —         | —     | —     | —     | —     | —     | —     | .0001 | .0003 | .0009 | .0019 | .0038 | .0070 | .0117 | .0181 |
| 29 | —         | —     | —     | —     | —     | —     | —     | .0001 | .0002 | .0004 | .0011 | .0023 | .0044 | .0077 | .0125 |
| 30 | —         | —     | —     | —     | —     | —     | —     | —     | .0001 | .0002 | .0006 | .0013 | .0026 | .0049 | .0083 |
| 31 | —         | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0003 | .0007 | .0015 | .0030 | .0054 |
| 32 | —         | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0001 | .0004 | .0009 | .0018 | .0034 |
| 33 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0002 | .0005 | .0010 | .0020 |
| 34 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0002 | .0006 | .0012 |
| 35 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0003 | .0007 |
| 36 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0002 | .0004 |
| 37 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 | .0002 |
| 38 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 |
| 39 | —         | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | .0001 |

# APPENDIX

# C-1

## STANDARD NORMAL AREAS

Example:  $P(0 < z < 1.96) = .4750$



This table shows the normal area between 0 and z.

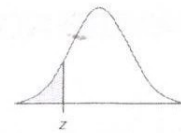
| z   | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | .0000  | .0040  | .0080  | .0120  | .0160  | .0199  | .0239  | .0279  | .0319  | .0359  |
| 0.1 | .0398  | .0438  | .0478  | .0517  | .0557  | .0596  | .0636  | .0675  | .0714  | .0753  |
| 0.2 | .0793  | .0832  | .0871  | .0910  | .0948  | .0987  | .1026  | .1064  | .1103  | .1141  |
| 0.3 | .1179  | .1217  | .1255  | .1293  | .1331  | .1368  | .1406  | .1443  | .1480  | .1517  |
| 0.4 | .1554  | .1591  | .1628  | .1664  | .1700  | .1736  | .1772  | .1808  | .1844  | .1879  |
| 0.5 | .1915  | .1950  | .1985  | .2019  | .2054  | .2088  | .2123  | .2157  | .2190  | .2224  |
| 0.6 | .2257  | .2291  | .2324  | .2357  | .2389  | .2422  | .2454  | .2486  | .2517  | .2549  |
| 0.7 | .2580  | .2611  | .2642  | .2673  | .2704  | .2734  | .2764  | .2794  | .2823  | .2852  |
| 0.8 | .2881  | .2910  | .2939  | .2967  | .2995  | .3023  | .3051  | .3078  | .3106  | .3133  |
| 0.9 | .3159  | .3186  | .3212  | .3238  | .3264  | .3289  | .3315  | .3340  | .3365  | .3389  |
| 1.0 | .3413  | .3438  | .3461  | .3485  | .3508  | .3531  | .3554  | .3577  | .3599  | .3621  |
| 1.1 | .3643  | .3665  | .3686  | .3708  | .3729  | .3749  | .3770  | .3790  | .3810  | .3830  |
| 1.2 | .3849  | .3869  | .3888  | .3907  | .3925  | .3944  | .3962  | .3980  | .3997  | .4015  |
| 1.3 | .4032  | .4049  | .4066  | .4082  | .4099  | .4115  | .4131  | .4147  | .4162  | .4177  |
| 1.4 | .4192  | .4207  | .4222  | .4236  | .4251  | .4265  | .4279  | .4292  | .4306  | .4319  |
| 1.5 | .4332  | .4345  | .4357  | .4370  | .4382  | .4394  | .4406  | .4418  | .4429  | .4441  |
| 1.6 | .4452  | .4463  | .4474  | .4484  | .4495  | .4505  | .4515  | .4525  | .4535  | .4545  |
| 1.7 | .4554  | .4564  | .4573  | .4582  | .4591  | .4599  | .4608  | .4616  | .4625  | .4633  |
| 1.8 | .4641  | .4649  | .4656  | .4664  | .4671  | .4678  | .4686  | .4693  | .4699  | .4706  |
| 1.9 | .4713  | .4719  | .4726  | .4732  | .4738  | .4744  | .4750  | .4756  | .4761  | .4767  |
| 2.0 | .4772  | .4778  | .4783  | .4788  | .4793  | .4798  | .4803  | .4808  | .4812  | .4817  |
| 2.1 | .4821  | .4826  | .4830  | .4834  | .4838  | .4842  | .4846  | .4850  | .4854  | .4857  |
| 2.2 | .4861  | .4864  | .4868  | .4871  | .4875  | .4878  | .4881  | .4884  | .4887  | .4890  |
| 2.3 | .4893  | .4896  | .4898  | .4901  | .4904  | .4906  | .4909  | .4911  | .4913  | .4916  |
| 2.4 | .4918  | .4920  | .4922  | .4925  | .4927  | .4929  | .4931  | .4932  | .4934  | .4936  |
| 2.5 | .4938  | .4940  | .4941  | .4943  | .4945  | .4946  | .4948  | .4949  | .4951  | .4952  |
| 2.6 | .4953  | .4955  | .4956  | .4957  | .4959  | .4960  | .4961  | .4962  | .4963  | .4964  |
| 2.7 | .4965  | .4966  | .4967  | .4968  | .4969  | .4970  | .4971  | .4972  | .4973  | .4974  |
| 2.8 | .4974  | .4975  | .4976  | .4977  | .4977  | .4978  | .4979  | .4979  | .4980  | .4981  |
| 2.9 | .4981  | .4982  | .4982  | .4983  | .4984  | .4984  | .4985  | .4985  | .4986  | .4986  |
| 3.0 | .49865 | .49869 | .49874 | .49878 | .49882 | .49886 | .49889 | .49893 | .49896 | .49900 |
| 3.1 | .49903 | .49906 | .49910 | .49913 | .49916 | .49918 | .49921 | .49924 | .49926 | .49929 |
| 3.2 | .49931 | .49934 | .49936 | .49938 | .49940 | .49942 | .49944 | .49946 | .49948 | .49950 |
| 3.3 | .49952 | .49953 | .49955 | .49957 | .49958 | .49960 | .49961 | .49962 | .49964 | .49965 |
| 3.4 | .49966 | .49968 | .49969 | .49970 | .49971 | .49972 | .49973 | .49974 | .49975 | .49976 |
| 3.5 | .49977 | .49978 | .49978 | .49979 | .49980 | .49981 | .49981 | .49982 | .49983 | .49983 |
| 3.6 | .49984 | .49985 | .49985 | .49986 | .49986 | .49987 | .49987 | .49988 | .49988 | .49989 |
| 3.7 | .49989 | .49990 | .49990 | .49990 | .49991 | .49991 | .49992 | .49992 | .49992 | .49992 |

# APPENDIX

# C-2

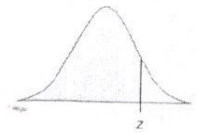
## CUMULATIVE STANDARD NORMAL DISTRIBUTION

Example:  $P(z < -1.96) = .0250$



This table shows the normal area less than  $z$ .

| $z$  | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -3.7 | .00011 | .00010 | .00010 | .00010 | .00009 | .00009 | .00008 | .00008 | .00008 | .00008 |
| -3.6 | .00016 | .00015 | .00015 | .00014 | .00014 | .00013 | .00013 | .00012 | .00012 | .00011 |
| -3.5 | .00023 | .00022 | .00022 | .00021 | .00020 | .00019 | .00019 | .00018 | .00017 | .00017 |
| -3.4 | .00034 | .00032 | .00031 | .00030 | .00029 | .00028 | .00027 | .00026 | .00025 | .00024 |
| -3.3 | .00048 | .00047 | .00045 | .00043 | .00042 | .00040 | .00039 | .00038 | .00036 | .00035 |
| -3.2 | .00069 | .00066 | .00064 | .00062 | .00060 | .00058 | .00056 | .00054 | .00052 | .00050 |
| -3.1 | .00097 | .00094 | .00090 | .00087 | .00084 | .00082 | .00079 | .00076 | .00074 | .00071 |
| -3.0 | .00135 | .00131 | .00126 | .00122 | .00118 | .00114 | .00111 | .00107 | .00104 | .00100 |
| -2.9 | .0019  | .0018  | .0018  | .0017  | .0016  | .0016  | .0015  | .0015  | .0014  | .0014  |
| -2.8 | .0026  | .0025  | .0024  | .0023  | .0023  | .0022  | .0021  | .0021  | .0020  | .0019  |
| -2.7 | .0035  | .0034  | .0033  | .0032  | .0031  | .0030  | .0029  | .0028  | .0027  | .0026  |
| -2.6 | .0047  | .0045  | .0044  | .0043  | .0041  | .0040  | .0039  | .0038  | .0037  | .0036  |
| -2.5 | .0062  | .0060  | .0059  | .0057  | .0055  | .0054  | .0052  | .0051  | .0049  | .0048  |
| -2.4 | .0082  | .0080  | .0078  | .0075  | .0073  | .0071  | .0069  | .0068  | .0066  | .0064  |
| -2.3 | .0107  | .0104  | .0102  | .0099  | .0096  | .0094  | .0091  | .0089  | .0087  | .0084  |
| -2.2 | .0139  | .0136  | .0132  | .0129  | .0125  | .0122  | .0119  | .0116  | .0113  | .0110  |
| -2.1 | .0179  | .0174  | .0170  | .0166  | .0162  | .0158  | .0154  | .0150  | .0146  | .0143  |
| -2.0 | .0228  | .0222  | .0217  | .0212  | .0207  | .0202  | .0197  | .0192  | .0188  | .0183  |
| -1.9 | .0287  | .0281  | .0274  | .0268  | .0262  | .0256  | .0250  | .0244  | .0239  | .0233  |
| -1.8 | .0359  | .0351  | .0344  | .0336  | .0329  | .0322  | .0314  | .0307  | .0301  | .0294  |
| -1.7 | .0446  | .0436  | .0427  | .0418  | .0409  | .0401  | .0392  | .0384  | .0375  | .0367  |
| -1.6 | .0548  | .0537  | .0526  | .0516  | .0505  | .0495  | .0485  | .0475  | .0465  | .0455  |
| -1.5 | .0668  | .0655  | .0643  | .0630  | .0618  | .0606  | .0594  | .0582  | .0571  | .0559  |
| -1.4 | .0808  | .0793  | .0778  | .0764  | .0749  | .0735  | .0721  | .0708  | .0694  | .0681  |
| -1.3 | .0968  | .0951  | .0934  | .0918  | .0901  | .0885  | .0869  | .0853  | .0838  | .0823  |
| -1.2 | .1151  | .1131  | .1112  | .1093  | .1075  | .1056  | .1038  | .1020  | .1003  | .0985  |
| -1.1 | .1357  | .1335  | .1314  | .1292  | .1271  | .1251  | .1230  | .1210  | .1190  | .1170  |
| -1.0 | .1587  | .1562  | .1539  | .1515  | .1492  | .1469  | .1446  | .1423  | .1401  | .1379  |
| -0.9 | .1841  | .1814  | .1788  | .1762  | .1736  | .1711  | .1685  | .1660  | .1635  | .1611  |
| -0.8 | .2119  | .2090  | .2061  | .2033  | .2005  | .1977  | .1949  | .1922  | .1894  | .1867  |
| -0.7 | .2420  | .2389  | .2358  | .2327  | .2296  | .2266  | .2236  | .2206  | .2177  | .2148  |
| -0.6 | .2743  | .2709  | .2676  | .2643  | .2611  | .2578  | .2546  | .2514  | .2483  | .2451  |
| -0.5 | .3085  | .3050  | .3015  | .2981  | .2946  | .2912  | .2877  | .2843  | .2810  | .2776  |
| -0.4 | .3446  | .3409  | .3372  | .3336  | .3300  | .3264  | .3228  | .3192  | .3156  | .3121  |
| -0.3 | .3821  | .3783  | .3745  | .3707  | .3669  | .3632  | .3594  | .3557  | .3520  | .3483  |
| -0.2 | .4207  | .4168  | .4129  | .4090  | .4052  | .4013  | .3974  | .3936  | .3897  | .3859  |
| -0.1 | .4602  | .4562  | .4522  | .4483  | .4443  | .4404  | .4364  | .4325  | .4286  | .4247  |
| -0.0 | .5000  | .4960  | .4920  | .4880  | .4841  | .4801  | .4761  | .4721  | .4681  | .4641  |



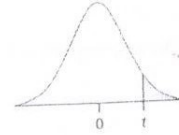
This table shows the normal area less than  $z$ .

| $z$ | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | .5000  | .5040  | .5080  | .5120  | .5160  | .5199  | .5239  | .5279  | .5319  | .5359  |
| 0.1 | .5398  | .5438  | .5478  | .5517  | .5557  | .5596  | .5636  | .5675  | .5714  | .5753  |
| 0.2 | .5793  | .5832  | .5871  | .5910  | .5948  | .5987  | .6026  | .6064  | .6103  | .6141  |
| 0.3 | .6179  | .6217  | .6255  | .6293  | .6331  | .6368  | .6406  | .6443  | .6480  | .6517  |
| 0.4 | .6554  | .6591  | .6628  | .6664  | .6700  | .6736  | .6772  | .6808  | .6844  | .6879  |
| 0.5 | .6915  | .6950  | .6985  | .7019  | .7054  | .7088  | .7123  | .7157  | .7190  | .7224  |
| 0.6 | .7257  | .7291  | .7324  | .7357  | .7389  | .7422  | .7454  | .7486  | .7517  | .7549  |
| 0.7 | .7580  | .7611  | .7642  | .7673  | .7704  | .7734  | .7764  | .7794  | .7823  | .7852  |
| 0.8 | .7881  | .7910  | .7939  | .7967  | .7995  | .8023  | .8051  | .8078  | .8106  | .8133  |
| 0.9 | .8159  | .8186  | .8212  | .8238  | .8264  | .8289  | .8315  | .8340  | .8365  | .8389  |
| 1.0 | .8413  | .8438  | .8461  | .8485  | .8508  | .8531  | .8554  | .8577  | .8599  | .8621  |
| 1.1 | .8643  | .8665  | .8686  | .8708  | .8729  | .8749  | .8770  | .8790  | .8810  | .8830  |
| 1.2 | .8849  | .8869  | .8888  | .8907  | .8925  | .8944  | .8962  | .8980  | .8997  | .9015  |
| 1.3 | .9032  | .9049  | .9066  | .9082  | .9099  | .9115  | .9131  | .9147  | .9162  | .9177  |
| 1.4 | .9192  | .9207  | .9222  | .9236  | .9251  | .9265  | .9279  | .9292  | .9306  | .9319  |
| 1.5 | .9332  | .9345  | .9357  | .9370  | .9382  | .9394  | .9406  | .9418  | .9429  | .9441  |
| 1.6 | .9452  | .9463  | .9474  | .9484  | .9495  | .9505  | .9515  | .9525  | .9535  | .9545  |
| 1.7 | .9554  | .9564  | .9573  | .9582  | .9591  | .9599  | .9608  | .9616  | .9625  | .9633  |
| 1.8 | .9641  | .9649  | .9656  | .9664  | .9671  | .9678  | .9686  | .9693  | .9699  | .9706  |
| 1.9 | .9713  | .9719  | .9726  | .9732  | .9738  | .9744  | .9750  | .9756  | .9761  | .9767  |
| 2.0 | .9772  | .9778  | .9783  | .9788  | .9793  | .9798  | .9803  | .9808  | .9812  | .9817  |
| 2.1 | .9821  | .9826  | .9830  | .9834  | .9838  | .9842  | .9846  | .9850  | .9854  | .9857  |
| 2.2 | .9861  | .9864  | .9868  | .9871  | .9875  | .9878  | .9881  | .9884  | .9887  | .9890  |
| 2.3 | .9893  | .9896  | .9898  | .9901  | .9904  | .9906  | .9909  | .9911  | .9913  | .9916  |
| 2.4 | .9918  | .9920  | .9922  | .9925  | .9927  | .9929  | .9931  | .9932  | .9934  | .9936  |
| 2.5 | .9938  | .9940  | .9941  | .9943  | .9945  | .9946  | .9948  | .9949  | .9951  | .9952  |
| 2.6 | .9953  | .9955  | .9956  | .9957  | .9959  | .9960  | .9961  | .9962  | .9963  | .9964  |
| 2.7 | .9965  | .9966  | .9967  | .9968  | .9969  | .9970  | .9971  | .9972  | .9973  | .9974  |
| 2.8 | .9974  | .9975  | .9976  | .9977  | .9977  | .9978  | .9979  | .9979  | .9980  | .9981  |
| 2.9 | .9981  | .9982  | .9982  | .9983  | .9984  | .9984  | .9985  | .9985  | .9986  | .9986  |
| 3.0 | .99865 | .99869 | .99874 | .99878 | .99882 | .99886 | .99889 | .99893 | .99896 | .99900 |
| 3.1 | .99903 | .99906 | .99910 | .99913 | .99916 | .99918 | .99921 | .99924 | .99926 | .99929 |
| 3.2 | .99931 | .99934 | .99936 | .99938 | .99940 | .99942 | .99944 | .99946 | .99948 | .99950 |
| 3.3 | .99952 | .99953 | .99955 | .99957 | .99958 | .99960 | .99961 | .99962 | .99964 | .99965 |
| 3.4 | .99966 | .99968 | .99969 | .99970 | .99971 | .99972 | .99973 | .99974 | .99975 | .99976 |
| 3.5 | .99977 | .99978 | .99978 | .99979 | .99980 | .99981 | .99981 | .99982 | .99983 | .99983 |
| 3.6 | .99984 | .99985 | .99985 | .99986 | .99986 | .99987 | .99987 | .99988 | .99988 | .99989 |
| 3.7 | .99989 | .99990 | .99990 | .99990 | .99991 | .99991 | .99992 | .99992 | .99992 | .99992 |

# APPENDIX

# D

## STUDENT'S *t* CRITICAL VALUES



This table shows the *t*-value that defines the area for the stated degrees of freedom (*v*).

| <i>v</i> | Confidence Level                       |       |        |        |        | <i>v</i> | Confidence Level                       |       |       |       |       |
|----------|--|-------|--------|--------|--------|----------|--|-------|-------|-------|-------|
|          | .80                                    | .90   | .95    | .98    | .99    |          | .80                                    | .90   | .95   | .98   | .99   |
|          | Significance Level for Two-Tailed Test |       |        |        |        |          | Significance Level for Two-Tailed Test |       |       |       |       |
|          | .20                                    | .10   | .05    | .02    | .01    |          | .20                                    | .10   | .05   | .02   | .01   |
|          | Significance Level for One-Tailed Test |       |        |        |        |          | Significance Level for One-Tailed Test |       |       |       |       |
|          | .10                                    | .05   | .025   | .01    | .005   |          | .10                                    | .05   | .025  | .01   | .005  |
| 1        | 3.078                                  | 6.314 | 12.706 | 31.821 | 63.656 | 36       | 1.306                                  | 1.688 | 2.028 | 2.434 | 2.719 |
| 2        | 1.886                                  | 2.920 | 4.303  | 6.965  | 9.925  | 37       | 1.305                                  | 1.687 | 2.026 | 2.431 | 2.715 |
| 3        | 1.638                                  | 2.353 | 3.182  | 4.541  | 5.841  | 38       | 1.304                                  | 1.686 | 2.024 | 2.429 | 2.712 |
| 4        | 1.533                                  | 2.132 | 2.776  | 3.747  | 4.604  | 39       | 1.304                                  | 1.685 | 2.023 | 2.426 | 2.708 |
| 5        | 1.476                                  | 2.015 | 2.571  | 3.365  | 4.032  | 40       | 1.303                                  | 1.684 | 2.021 | 2.423 | 2.704 |
| 6        | 1.440                                  | 1.943 | 2.447  | 3.143  | 3.707  | 41       | 1.303                                  | 1.683 | 2.020 | 2.421 | 2.701 |
| 7        | 1.415                                  | 1.895 | 2.365  | 2.998  | 3.499  | 42       | 1.302                                  | 1.682 | 2.018 | 2.418 | 2.698 |
| 8        | 1.397                                  | 1.860 | 2.306  | 2.896  | 3.355  | 43       | 1.302                                  | 1.681 | 2.017 | 2.416 | 2.695 |
| 9        | 1.383                                  | 1.833 | 2.262  | 2.821  | 3.250  | 44       | 1.301                                  | 1.680 | 2.015 | 2.414 | 2.692 |
| 10       | 1.372                                  | 1.812 | 2.228  | 2.764  | 3.169  | 45       | 1.301                                  | 1.679 | 2.014 | 2.412 | 2.690 |
| 11       | 1.363                                  | 1.796 | 2.201  | 2.718  | 3.106  | 46       | 1.300                                  | 1.679 | 2.013 | 2.410 | 2.687 |
| 12       | 1.356                                  | 1.782 | 2.179  | 2.681  | 3.055  | 47       | 1.300                                  | 1.678 | 2.012 | 2.408 | 2.685 |
| 13       | 1.350                                  | 1.771 | 2.160  | 2.650  | 3.012  | 48       | 1.299                                  | 1.677 | 2.011 | 2.407 | 2.682 |
| 14       | 1.345                                  | 1.761 | 2.145  | 2.624  | 2.977  | 49       | 1.299                                  | 1.677 | 2.010 | 2.405 | 2.680 |
| 15       | 1.341                                  | 1.753 | 2.131  | 2.602  | 2.947  | 50       | 1.299                                  | 1.676 | 2.009 | 2.403 | 2.678 |
| 16       | 1.337                                  | 1.746 | 2.120  | 2.583  | 2.921  | 55       | 1.297                                  | 1.673 | 2.004 | 2.396 | 2.668 |
| 17       | 1.333                                  | 1.740 | 2.110  | 2.567  | 2.898  | 60       | 1.296                                  | 1.671 | 2.000 | 2.390 | 2.660 |
| 18       | 1.330                                  | 1.734 | 2.101  | 2.552  | 2.878  | 65       | 1.295                                  | 1.669 | 1.997 | 2.385 | 2.654 |
| 19       | 1.328                                  | 1.729 | 2.093  | 2.539  | 2.861  | 70       | 1.294                                  | 1.667 | 1.994 | 2.381 | 2.648 |
| 20       | 1.325                                  | 1.725 | 2.086  | 2.528  | 2.845  | 75       | 1.293                                  | 1.665 | 1.992 | 2.377 | 2.643 |
| 21       | 1.323                                  | 1.721 | 2.080  | 2.518  | 2.831  | 80       | 1.292                                  | 1.664 | 1.990 | 2.374 | 2.639 |
| 22       | 1.321                                  | 1.717 | 2.074  | 2.508  | 2.819  | 85       | 1.292                                  | 1.663 | 1.988 | 2.371 | 2.635 |
| 23       | 1.319                                  | 1.714 | 2.069  | 2.500  | 2.807  | 90       | 1.291                                  | 1.662 | 1.987 | 2.368 | 2.632 |
| 24       | 1.318                                  | 1.711 | 2.064  | 2.492  | 2.797  | 95       | 1.291                                  | 1.661 | 1.985 | 2.366 | 2.629 |
| 25       | 1.316                                  | 1.708 | 2.060  | 2.485  | 2.787  | 100      | 1.290                                  | 1.660 | 1.984 | 2.364 | 2.626 |
| 26       | 1.315                                  | 1.706 | 2.056  | 2.479  | 2.779  | 110      | 1.289                                  | 1.659 | 1.982 | 2.361 | 2.621 |
| 27       | 1.314                                  | 1.703 | 2.052  | 2.473  | 2.771  | 120      | 1.289                                  | 1.658 | 1.980 | 2.358 | 2.617 |
| 28       | 1.313                                  | 1.701 | 2.048  | 2.467  | 2.763  | 130      | 1.288                                  | 1.657 | 1.978 | 2.355 | 2.614 |
| 29       | 1.311                                  | 1.699 | 2.045  | 2.462  | 2.756  | 140      | 1.288                                  | 1.656 | 1.977 | 2.353 | 2.611 |
| 30       | 1.310                                  | 1.697 | 2.042  | 2.457  | 2.750  | 150      | 1.287                                  | 1.655 | 1.976 | 2.351 | 2.609 |
| 31       | 1.309                                  | 1.696 | 2.040  | 2.453  | 2.744  | ∞        | 1.282                                  | 1.645 | 1.960 | 2.326 | 2.576 |
| 32       | 1.309                                  | 1.694 | 2.037  | 2.449  | 2.738  |          |  |       |       |       |       |
| 33       | 1.308                                  | 1.692 | 2.035  | 2.445  | 2.733  |          |  |       |       |       |       |
| 34       | 1.307                                  | 1.691 | 2.032  | 2.441  | 2.728  |          |  |       |       |       |       |
| 35       | 1.306                                  | 1.690 | 2.030  | 2.438  | 2.724  |          |  |       |       |       |       |

Note: As *n* increases, critical values of Student's *t* approach the *z*-values in the last line of this table. A common rule of thumb is to use *z* when *n* > 30, but that is *not* conservative.

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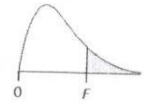
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# APPENDIX

# F

## CRITICAL VALUES OF $F_{.10}$

This table shows the 10 percent right-tail critical values of  $F$  for the stated degrees of freedom ( $\nu$ ).



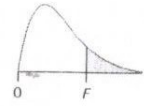
| Denominator<br>Degrees of<br>Freedom ( $\nu_2$ ) | Numerator Degrees of Freedom ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |       |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|  | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    |
| 1  | 39.86                                    | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 | 60.19 | 60.71 |
| 2  | 8.53                                     | 9.00  | 9.16  | 9.24  | 9.29  | 9.33  | 9.35  | 9.37  | 9.38  | 9.39  | 9.41  |
| 3  | 5.54                                     | 5.46  | 5.39  | 5.34  | 5.31  | 5.28  | 5.27  | 5.25  | 5.24  | 5.23  | 5.22  |
| 4  | 4.54                                     | 4.32  | 4.19  | 4.11  | 4.05  | 4.01  | 3.98  | 3.95  | 3.94  | 3.92  | 3.90  |
| 5  | 4.06                                     | 3.78  | 3.62  | 3.52  | 3.45  | 3.40  | 3.37  | 3.34  | 3.32  | 3.30  | 3.27  |
| 6  | 3.78                                     | 3.46  | 3.29  | 3.18  | 3.11  | 3.05  | 3.01  | 2.98  | 2.96  | 2.94  | 2.90  |
| 7  | 3.59                                     | 3.26  | 3.07  | 2.96  | 2.88  | 2.83  | 2.78  | 2.75  | 2.72  | 2.70  | 2.67  |
| 8  | 3.46                                     | 3.11  | 2.92  | 2.81  | 2.73  | 2.67  | 2.62  | 2.59  | 2.56  | 2.54  | 2.50  |
| 9  | 3.36                                     | 3.01  | 2.81  | 2.69  | 2.61  | 2.55  | 2.51  | 2.47  | 2.44  | 2.42  | 2.38  |
| 10   | 3.29                                     | 2.92  | 2.73  | 2.61  | 2.52  | 2.46  | 2.41  | 2.38  | 2.35  | 2.32  | 2.28  |
| 11   | 3.23                                     | 2.86  | 2.66  | 2.54  | 2.45  | 2.39  | 2.34  | 2.30  | 2.27  | 2.25  | 2.21  |
| 12   | 3.18                                     | 2.81  | 2.61  | 2.48  | 2.39  | 2.33  | 2.28  | 2.24  | 2.21  | 2.19  | 2.15  |
| 13   | 3.14                                     | 2.76  | 2.56  | 2.43  | 2.35  | 2.28  | 2.23  | 2.20  | 2.16  | 2.14  | 2.10  |
| 14   | 3.10                                     | 2.73  | 2.52  | 2.39  | 2.31  | 2.24  | 2.19  | 2.15  | 2.12  | 2.10  | 2.05  |
| 15   | 3.07                                     | 2.70  | 2.49  | 2.36  | 2.27  | 2.21  | 2.16  | 2.12  | 2.09  | 2.06  | 2.02  |
| 16   | 3.05                                     | 2.67  | 2.46  | 2.33  | 2.24  | 2.18  | 2.13  | 2.09  | 2.06  | 2.03  | 1.99  |
| 17   | 3.03                                     | 2.64  | 2.44  | 2.31  | 2.22  | 2.15  | 2.10  | 2.06  | 2.03  | 2.00  | 1.96  |
| 18   | 3.01                                     | 2.62  | 2.42  | 2.29  | 2.20  | 2.13  | 2.08  | 2.04  | 2.00  | 1.98  | 1.93  |
| 19   | 2.99                                     | 2.61  | 2.40  | 2.27  | 2.18  | 2.11  | 2.06  | 2.02  | 1.98  | 1.96  | 1.91  |
| 20   | 2.97                                     | 2.59  | 2.38  | 2.25  | 2.16  | 2.09  | 2.04  | 2.00  | 1.96  | 1.94  | 1.89  |
| 21   | 2.96                                     | 2.57  | 2.36  | 2.23  | 2.14  | 2.08  | 2.02  | 1.98  | 1.95  | 1.92  | 1.87  |
| 22   | 2.95                                     | 2.56  | 2.35  | 2.22  | 2.13  | 2.06  | 2.01  | 1.97  | 1.93  | 1.90  | 1.86  |
| 23   | 2.94                                     | 2.55  | 2.34  | 2.21  | 2.11  | 2.05  | 1.99  | 1.95  | 1.92  | 1.89  | 1.84  |
| 24   | 2.93                                     | 2.54  | 2.33  | 2.19  | 2.10  | 2.04  | 1.98  | 1.94  | 1.91  | 1.88  | 1.83  |
| 25   | 2.92                                     | 2.53  | 2.32  | 2.18  | 2.09  | 2.02  | 1.97  | 1.93  | 1.89  | 1.87  | 1.82  |
| 26   | 2.91                                     | 2.52  | 2.31  | 2.17  | 2.08  | 2.01  | 1.96  | 1.92  | 1.88  | 1.86  | 1.81  |
| 27   | 2.90                                     | 2.51  | 2.30  | 2.17  | 2.07  | 2.00  | 1.95  | 1.91  | 1.87  | 1.85  | 1.80  |
| 28   | 2.89                                     | 2.50  | 2.29  | 2.16  | 2.06  | 2.00  | 1.94  | 1.90  | 1.87  | 1.84  | 1.79  |
| 29   | 2.89                                     | 2.50  | 2.28  | 2.15  | 2.06  | 1.99  | 1.93  | 1.89  | 1.86  | 1.83  | 1.78  |
| 30   | 2.88                                     | 2.49  | 2.28  | 2.14  | 2.05  | 1.98  | 1.93  | 1.88  | 1.85  | 1.82  | 1.77  |
| 40   | 2.84                                     | 2.44  | 2.23  | 2.09  | 2.00  | 1.93  | 1.87  | 1.83  | 1.79  | 1.76  | 1.71  |
| 50   | 2.81                                     | 2.41  | 2.20  | 2.06  | 1.97  | 1.90  | 1.84  | 1.80  | 1.76  | 1.73  | 1.68  |
| 60   | 2.79                                     | 2.39  | 2.18  | 2.04  | 1.95  | 1.87  | 1.82  | 1.77  | 1.74  | 1.71  | 1.66  |
| 120  | 2.75                                     | 2.35  | 2.13  | 1.99  | 1.90  | 1.82  | 1.77  | 1.72  | 1.68  | 1.65  | 1.60  |
| 200  | 2.73                                     | 2.33  | 2.11  | 1.97  | 1.88  | 1.80  | 1.75  | 1.70  | 1.66  | 1.63  | 1.58  |
| $\infty$   | 2.71                                     | 2.30  | 2.08  | 1.94  | 1.85  | 1.77  | 1.72  | 1.67  | 1.63  | 1.60  | 1.55  |



| Denominator<br>Degrees of<br>Freedom ( $\nu_2$ ) | Numerator Degrees of Freedom ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |          |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|  | 15                                       | 20    | 25    | 30    | 35    | 40    | 50    | 60    | 120   | 200   | $\infty$ |
| 1  | 61.22                                    | 61.74 | 62.05 | 62.26 | 62.42 | 62.53 | 62.69 | 62.79 | 63.06 | 63.17 | 63.32    |
| 2  | 9.42                                     | 9.44  | 9.45  | 9.46  | 9.46  | 9.47  | 9.47  | 9.47  | 9.48  | 9.49  | 9.49     |
| 3  | 5.20                                     | 5.18  | 5.17  | 5.17  | 5.16  | 5.16  | 5.15  | 5.15  | 5.14  | 5.14  | 5.13     |
| 4  | 3.87                                     | 3.84  | 3.83  | 3.82  | 3.81  | 3.80  | 3.80  | 3.79  | 3.78  | 3.77  | 3.76     |
| 5  | 3.24                                     | 3.21  | 3.19  | 3.17  | 3.16  | 3.16  | 3.15  | 3.14  | 3.12  | 3.12  | 3.11     |
| 6  | 2.87                                     | 2.84  | 2.81  | 2.80  | 2.79  | 2.78  | 2.77  | 2.76  | 2.74  | 2.73  | 2.72     |
| 7  | 2.63                                     | 2.59  | 2.57  | 2.56  | 2.54  | 2.54  | 2.52  | 2.51  | 2.49  | 2.48  | 2.47     |
| 8  | 2.46                                     | 2.42  | 2.40  | 2.38  | 2.37  | 2.36  | 2.35  | 2.34  | 2.32  | 2.31  | 2.29     |
| 9  | 2.34                                     | 2.30  | 2.27  | 2.25  | 2.24  | 2.23  | 2.22  | 2.21  | 2.18  | 2.17  | 2.16     |
| 10   | 2.24                                     | 2.20  | 2.17  | 2.16  | 2.14  | 2.13  | 2.12  | 2.11  | 2.08  | 2.07  | 2.06     |
| 11   | 2.17                                     | 2.12  | 2.10  | 2.08  | 2.06  | 2.05  | 2.04  | 2.03  | 2.00  | 1.99  | 1.97     |
| 12   | 2.10                                     | 2.06  | 2.03  | 2.01  | 2.00  | 1.99  | 1.97  | 1.96  | 1.93  | 1.92  | 1.90     |
| 13   | 2.05                                     | 2.01  | 1.98  | 1.96  | 1.94  | 1.93  | 1.92  | 1.90  | 1.88  | 1.86  | 1.85     |
| 14   | 2.01                                     | 1.96  | 1.93  | 1.91  | 1.90  | 1.89  | 1.87  | 1.86  | 1.83  | 1.82  | 1.80     |
| 15   | 1.97                                     | 1.92  | 1.89  | 1.87  | 1.86  | 1.85  | 1.83  | 1.82  | 1.79  | 1.77  | 1.76     |
| 16   | 1.94                                     | 1.89  | 1.86  | 1.84  | 1.82  | 1.81  | 1.79  | 1.78  | 1.75  | 1.74  | 1.72     |
| 17   | 1.91                                     | 1.86  | 1.83  | 1.81  | 1.79  | 1.78  | 1.76  | 1.75  | 1.72  | 1.71  | 1.69     |
| 18   | 1.89                                     | 1.84  | 1.80  | 1.78  | 1.77  | 1.75  | 1.74  | 1.72  | 1.69  | 1.68  | 1.66     |
| 19   | 1.86                                     | 1.81  | 1.78  | 1.76  | 1.74  | 1.73  | 1.71  | 1.70  | 1.67  | 1.65  | 1.63     |
| 20   | 1.84                                     | 1.79  | 1.76  | 1.74  | 1.72  | 1.71  | 1.69  | 1.68  | 1.64  | 1.63  | 1.61     |
| 21   | 1.83                                     | 1.78  | 1.74  | 1.72  | 1.70  | 1.69  | 1.67  | 1.66  | 1.62  | 1.61  | 1.59     |
| 22   | 1.81                                     | 1.76  | 1.73  | 1.70  | 1.68  | 1.67  | 1.65  | 1.64  | 1.60  | 1.59  | 1.57     |
| 23   | 1.80                                     | 1.74  | 1.71  | 1.69  | 1.67  | 1.66  | 1.64  | 1.62  | 1.59  | 1.57  | 1.55     |
| 24   | 1.78                                     | 1.73  | 1.70  | 1.67  | 1.65  | 1.64  | 1.62  | 1.61  | 1.57  | 1.56  | 1.53     |
| 25   | 1.77                                     | 1.72  | 1.68  | 1.66  | 1.64  | 1.63  | 1.61  | 1.59  | 1.56  | 1.54  | 1.52     |
| 26   | 1.76                                     | 1.71  | 1.67  | 1.65  | 1.63  | 1.61  | 1.59  | 1.58  | 1.54  | 1.53  | 1.50     |
| 27   | 1.75                                     | 1.70  | 1.66  | 1.64  | 1.62  | 1.60  | 1.58  | 1.57  | 1.53  | 1.52  | 1.49     |
| 28   | 1.74                                     | 1.69  | 1.65  | 1.63  | 1.61  | 1.59  | 1.57  | 1.56  | 1.52  | 1.50  | 1.48     |
| 29   | 1.73                                     | 1.68  | 1.64  | 1.62  | 1.60  | 1.58  | 1.56  | 1.55  | 1.51  | 1.49  | 1.47     |
| 30   | 1.72                                     | 1.67  | 1.63  | 1.61  | 1.59  | 1.57  | 1.55  | 1.54  | 1.50  | 1.48  | 1.46     |
| 40   | 1.66                                     | 1.61  | 1.57  | 1.54  | 1.52  | 1.51  | 1.48  | 1.47  | 1.42  | 1.41  | 1.38     |
| 50   | 1.63                                     | 1.57  | 1.53  | 1.50  | 1.48  | 1.46  | 1.44  | 1.42  | 1.38  | 1.36  | 1.33     |
| 60   | 1.60                                     | 1.54  | 1.50  | 1.48  | 1.45  | 1.44  | 1.41  | 1.40  | 1.35  | 1.33  | 1.29     |
| 120  | 1.55                                     | 1.48  | 1.44  | 1.41  | 1.39  | 1.37  | 1.34  | 1.32  | 1.26  | 1.24  | 1.19     |
| 200  | 1.52                                     | 1.46  | 1.41  | 1.38  | 1.36  | 1.34  | 1.31  | 1.29  | 1.23  | 1.20  | 1.15     |
| $\infty$   | 2.71                                     | 1.49  | 1.42  | 1.38  | 1.34  | 1.32  | 1.30  | 1.26  | 1.24  | 1.17  | 1.13     |

# CRITICAL VALUES OF $F_{.05}$

This table shows the 5 percent right-tail critical values of  $F$  for the stated degrees of freedom ( $\nu$ ).

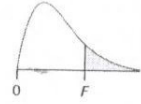


| Denominator<br>Degrees of<br>Freedom ( $\nu_2$ ) | Numerator Degrees of Freedom ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |       |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|  | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    |
| 1  | 161.4                                    | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 |
| 2  | 18.51                                    | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 |
| 3  | 10.13                                    | 9.55  | 9.28  | 9.12  | 9.01  | 8.94  | 8.89  | 8.85  | 8.81  | 8.79  | 8.74  |
| 4  | 7.71                                     | 6.94  | 6.59  | 6.39  | 6.26  | 6.16  | 6.09  | 6.04  | 6.00  | 5.96  | 5.91  |
| 5  | 6.61                                     | 5.79  | 5.41  | 5.19  | 5.05  | 4.95  | 4.88  | 4.82  | 4.77  | 4.74  | 4.68  |
| 6  | 5.99                                     | 5.14  | 4.76  | 4.53  | 4.39  | 4.28  | 4.21  | 4.15  | 4.10  | 4.06  | 4.00  |
| 7  | 5.59                                     | 4.74  | 4.35  | 4.12  | 3.97  | 3.87  | 3.79  | 3.73  | 3.68  | 3.64  | 3.57  |
| 8  | 5.32                                     | 4.46  | 4.07  | 3.84  | 3.69  | 3.58  | 3.50  | 3.44  | 3.39  | 3.35  | 3.28  |
| 9  | 5.12                                     | 4.26  | 3.86  | 3.63  | 3.48  | 3.37  | 3.29  | 3.23  | 3.18  | 3.14  | 3.07  |
| 10   | 4.96                                     | 4.10  | 3.71  | 3.48  | 3.33  | 3.22  | 3.14  | 3.07  | 3.02  | 2.98  | 2.91  |
| 11   | 4.84                                     | 3.98  | 3.59  | 3.36  | 3.20  | 3.09  | 3.01  | 2.95  | 2.90  | 2.85  | 2.79  |
| 12   | 4.75                                     | 3.89  | 3.49  | 3.26  | 3.11  | 3.00  | 2.91  | 2.85  | 2.80  | 2.75  | 2.69  |
| 13   | 4.67                                     | 3.81  | 3.41  | 3.18  | 3.03  | 2.92  | 2.83  | 2.77  | 2.71  | 2.67  | 2.60  |
| 14   | 4.60                                     | 3.74  | 3.34  | 3.11  | 2.96  | 2.85  | 2.76  | 2.70  | 2.65  | 2.60  | 2.53  |
| 15   | 4.54                                     | 3.68  | 3.29  | 3.06  | 2.90  | 2.79  | 2.71  | 2.64  | 2.59  | 2.54  | 2.48  |
| 16   | 4.49                                     | 3.63  | 3.24  | 3.01  | 2.85  | 2.74  | 2.66  | 2.59  | 2.54  | 2.49  | 2.42  |
| 17   | 4.45                                     | 3.59  | 3.20  | 2.96  | 2.81  | 2.70  | 2.61  | 2.55  | 2.49  | 2.45  | 2.38  |
| 18   | 4.41                                     | 3.55  | 3.16  | 2.93  | 2.77  | 2.66  | 2.58  | 2.51  | 2.46  | 2.41  | 2.34  |
| 19   | 4.38                                     | 3.52  | 3.13  | 2.90  | 2.74  | 2.63  | 2.54  | 2.48  | 2.42  | 2.38  | 2.31  |
| 20   | 4.35                                     | 3.49  | 3.10  | 2.87  | 2.71  | 2.60  | 2.51  | 2.45  | 2.39  | 2.35  | 2.28  |
| 21   | 4.32                                     | 3.47  | 3.07  | 2.84  | 2.68  | 2.57  | 2.49  | 2.42  | 2.37  | 2.32  | 2.25  |
| 22   | 4.30                                     | 3.44  | 3.05  | 2.82  | 2.66  | 2.55  | 2.46  | 2.40  | 2.34  | 2.30  | 2.23  |
| 23   | 4.28                                     | 3.42  | 3.03  | 2.80  | 2.64  | 2.53  | 2.44  | 2.37  | 2.32  | 2.27  | 2.20  |
| 24   | 4.26                                     | 3.40  | 3.01  | 2.78  | 2.62  | 2.51  | 2.42  | 2.36  | 2.30  | 2.25  | 2.18  |
| 25   | 4.24                                     | 3.39  | 2.99  | 2.76  | 2.60  | 2.49  | 2.40  | 2.34  | 2.28  | 2.24  | 2.16  |
| 26   | 4.23                                     | 3.37  | 2.98  | 2.74  | 2.59  | 2.47  | 2.39  | 2.32  | 2.27  | 2.22  | 2.15  |
| 27   | 4.21                                     | 3.35  | 2.96  | 2.73  | 2.57  | 2.46  | 2.37  | 2.31  | 2.25  | 2.20  | 2.13  |
| 28   | 4.20                                     | 3.34  | 2.95  | 2.71  | 2.56  | 2.45  | 2.36  | 2.29  | 2.24  | 2.19  | 2.12  |
| 29   | 4.18                                     | 3.33  | 2.93  | 2.70  | 2.55  | 2.43  | 2.35  | 2.28  | 2.22  | 2.18  | 2.10  |
| 30   | 4.17                                     | 3.32  | 2.92  | 2.69  | 2.53  | 2.42  | 2.33  | 2.27  | 2.21  | 2.16  | 2.09  |
| 40   | 4.08                                     | 3.23  | 2.84  | 2.61  | 2.45  | 2.34  | 2.25  | 2.18  | 2.12  | 2.08  | 2.00  |
| 50   | 4.03                                     | 3.18  | 2.79  | 2.56  | 2.40  | 2.29  | 2.20  | 2.13  | 2.07  | 2.03  | 1.95  |
| 60   | 4.00                                     | 3.15  | 2.76  | 2.53  | 2.37  | 2.25  | 2.17  | 2.10  | 2.04  | 1.99  | 1.92  |
| ∞  | 3.92                                     | 3.07  | 2.68  | 2.45  | 2.29  | 2.18  | 2.09  | 2.02  | 1.96  | 1.91  | 1.83  |
| 200  | 3.89                                     | 3.04  | 2.65  | 2.42  | 2.26  | 2.14  | 2.06  | 1.98  | 1.93  | 1.88  | 1.80  |
| ∞  | 2.71                                     | 3.84  | 3.00  | 2.60  | 2.37  | 2.21  | 2.10  | 2.01  | 1.94  | 1.88  | 1.83  |

| Denominator<br>Degrees of<br>Freedom ( $\nu_2$ ) | Numerator Degrees of Freedom ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |          |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|  | 15                                       | 20    | 25    | 30    | 35    | 40    | 50    | 60    | 120   | 200   | $\infty$ |
| 1  | 245.9                                    | 248.0 | 249.3 | 250.1 | 250.7 | 251.1 | 251.8 | 252.2 | 253.3 | 253.7 | 254.3    |
| 2  | 19.43                                    | 19.45 | 19.46 | 19.46 | 19.47 | 19.47 | 19.48 | 19.48 | 19.49 | 19.49 | 19.50    |
| 3  | 8.70                                     | 8.66  | 8.63  | 8.62  | 8.60  | 8.59  | 8.58  | 8.57  | 8.55  | 8.54  | 8.53     |
| 4  | 5.86                                     | 5.80  | 5.77  | 5.75  | 5.73  | 5.72  | 5.70  | 5.69  | 5.66  | 5.65  | 5.63     |
| 5  | 4.62                                     | 4.56  | 4.52  | 4.50  | 4.48  | 4.46  | 4.44  | 4.43  | 4.40  | 4.39  | 4.37     |
| 6  | 3.94                                     | 3.87  | 3.83  | 3.81  | 3.79  | 3.77  | 3.75  | 3.74  | 3.70  | 3.69  | 3.67     |
| 7  | 3.51                                     | 3.44  | 3.40  | 3.38  | 3.36  | 3.34  | 3.32  | 3.30  | 3.27  | 3.25  | 3.23     |
| 8  | 3.22                                     | 3.15  | 3.11  | 3.08  | 3.06  | 3.04  | 3.02  | 3.01  | 2.97  | 2.95  | 2.93     |
| 9  | 3.01                                     | 2.94  | 2.89  | 2.86  | 2.84  | 2.83  | 2.80  | 2.79  | 2.75  | 2.73  | 2.71     |
| 10   | 2.85                                     | 2.77  | 2.73  | 2.70  | 2.68  | 2.66  | 2.64  | 2.62  | 2.58  | 2.56  | 2.54     |
| 11   | 2.72                                     | 2.65  | 2.60  | 2.57  | 2.55  | 2.53  | 2.51  | 2.49  | 2.45  | 2.43  | 2.41     |
| 12   | 2.62                                     | 2.54  | 2.50  | 2.47  | 2.44  | 2.43  | 2.40  | 2.38  | 2.34  | 2.32  | 2.30     |
| 13   | 2.53                                     | 2.46  | 2.41  | 2.38  | 2.36  | 2.34  | 2.31  | 2.30  | 2.25  | 2.23  | 2.21     |
| 14   | 2.46                                     | 2.39  | 2.34  | 2.31  | 2.28  | 2.27  | 2.24  | 2.22  | 2.18  | 2.16  | 2.13     |
| 15   | 2.40                                     | 2.33  | 2.28  | 2.25  | 2.22  | 2.20  | 2.18  | 2.16  | 2.11  | 2.10  | 2.07     |
| 16   | 2.35                                     | 2.28  | 2.23  | 2.19  | 2.17  | 2.15  | 2.12  | 2.11  | 2.06  | 2.04  | 2.01     |
| 17   | 2.31                                     | 2.23  | 2.18  | 2.15  | 2.12  | 2.10  | 2.08  | 2.06  | 2.01  | 1.99  | 1.96     |
| 18   | 2.27                                     | 2.19  | 2.14  | 2.11  | 2.08  | 2.06  | 2.04  | 2.02  | 1.97  | 1.95  | 1.92     |
| 19   | 2.23                                     | 2.16  | 2.11  | 2.07  | 2.05  | 2.03  | 2.00  | 1.98  | 1.93  | 1.91  | 1.88     |
| 20   | 2.20                                     | 2.12  | 2.07  | 2.04  | 2.01  | 1.99  | 1.97  | 1.95  | 1.90  | 1.88  | 1.84     |
| 21   | 2.18                                     | 2.10  | 2.05  | 2.01  | 1.98  | 1.96  | 1.94  | 1.92  | 1.87  | 1.84  | 1.81     |
| 22   | 2.15                                     | 2.07  | 2.02  | 1.98  | 1.96  | 1.94  | 1.91  | 1.89  | 1.84  | 1.82  | 1.78     |
| 23   | 2.13                                     | 2.05  | 2.00  | 1.96  | 1.93  | 1.91  | 1.88  | 1.86  | 1.81  | 1.79  | 1.76     |
| 24   | 2.11                                     | 2.03  | 1.97  | 1.94  | 1.91  | 1.89  | 1.86  | 1.84  | 1.79  | 1.77  | 1.73     |
| 25   | 2.09                                     | 2.01  | 1.96  | 1.92  | 1.89  | 1.87  | 1.84  | 1.82  | 1.77  | 1.75  | 1.71     |
| 26   | 2.07                                     | 1.99  | 1.94  | 1.90  | 1.87  | 1.85* | 1.82  | 1.80  | 1.75  | 1.73  | 1.69     |
| 27   | 2.06                                     | 1.97  | 1.92  | 1.88  | 1.86  | 1.84  | 1.81  | 1.79  | 1.73  | 1.71  | 1.67     |
| 28   | 2.04                                     | 1.96  | 1.91  | 1.87  | 1.84  | 1.82  | 1.79  | 1.77  | 1.71  | 1.69  | 1.66     |
| 29   | 2.03                                     | 1.94  | 1.89  | 1.85  | 1.83  | 1.81  | 1.77  | 1.75  | 1.70  | 1.67  | 1.64     |
| 30   | 2.01                                     | 1.93  | 1.88  | 1.84  | 1.81  | 1.79  | 1.76  | 1.74  | 1.68  | 1.66  | 1.62     |
| 40   | 1.92                                     | 1.84  | 1.78  | 1.74  | 1.72  | 1.69  | 1.66  | 1.64  | 1.58  | 1.55  | 1.51     |
| 50   | 1.87                                     | 1.78  | 1.73  | 1.69  | 1.66  | 1.63  | 1.60  | 1.58  | 1.51  | 1.48  | 1.44     |
| 60   | 1.84                                     | 1.75  | 1.69  | 1.65  | 1.62  | 1.59  | 1.56  | 1.53  | 1.47  | 1.44  | 1.39     |
| 120  | 1.75                                     | 1.66  | 1.60  | 1.55  | 1.52  | 1.50  | 1.46  | 1.43  | 1.35  | 1.32  | 1.26     |
| 200  | 1.72                                     | 1.62  | 1.56  | 1.52  | 1.48  | 1.46  | 1.41  | 1.39  | 1.30  | 1.26  | 1.19     |
| $\infty$   | 2.71                                     | 1.67  | 1.57  | 1.51  | 1.46  | 1.42  | 1.39  | 1.35  | 1.32  | 1.22  | 1.17     |

CRITICAL VALUES OF  $F_{.025}$ 

This table shows the 2.5 percent right-tail critical values of  $F$  for the stated degrees of freedom ( $\nu$ ).

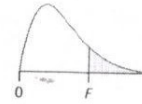


| Denominator<br>Degrees of<br>Freedom ( $\nu_2$ ) | Numerator Degrees of Freedom ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |       |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|  | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    |
| 1  | 647.8                                    | 799.5 | 864.2 | 899.6 | 921.8 | 937.1 | 948.2 | 956.6 | 963.3 | 968.6 | 976.7 |
| 2  | 38.51                                    | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.41 |
| 3  | 17.44                                    | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.34 |
| 4  | 12.22                                    | 10.65 | 9.98  | 9.60  | 9.36  | 9.20  | 9.07  | 8.98  | 8.90  | 8.84  | 8.75  |
| 5  | 10.01                                    | 8.43  | 7.76  | 7.39  | 7.15  | 6.98  | 6.85  | 6.76  | 6.68  | 6.62  | 6.52  |
| 6  | 8.81                                     | 7.26  | 6.60  | 6.23  | 5.99  | 5.82  | 5.70  | 5.60  | 5.52  | 5.46  | 5.37  |
| 7  | 8.07                                     | 6.54  | 5.89  | 5.52  | 5.29  | 5.12  | 4.99  | 4.90  | 4.82  | 4.76  | 4.67  |
| 8  | 7.57                                     | 6.06  | 5.42  | 5.05  | 4.82  | 4.65  | 4.53  | 4.43  | 4.36  | 4.30  | 4.20  |
| 9  | 7.21                                     | 5.71  | 5.08  | 4.72  | 4.48  | 4.32  | 4.20  | 4.10  | 4.03  | 3.96  | 3.87  |
| 10   | 6.94                                     | 5.46  | 4.83  | 4.47  | 4.24  | 4.07  | 3.95  | 3.85  | 3.78  | 3.72  | 3.62  |
| 11   | 6.72                                     | 5.26  | 4.63  | 4.28  | 4.04  | 3.88  | 3.76  | 3.66  | 3.59  | 3.53  | 3.43  |
| 12   | 6.55                                     | 5.10  | 4.47  | 4.12  | 3.89  | 3.73  | 3.61  | 3.51  | 3.44  | 3.37  | 3.28  |
| 13   | 6.41                                     | 4.97  | 4.35  | 4.00  | 3.77  | 3.60  | 3.48  | 3.39  | 3.31  | 3.25  | 3.15  |
| 14   | 6.30                                     | 4.86  | 4.24  | 3.89  | 3.66  | 3.50  | 3.38  | 3.29  | 3.21  | 3.15  | 3.05  |
| 15   | 6.20                                     | 4.77  | 4.15  | 3.80  | 3.58  | 3.41  | 3.29  | 3.20  | 3.12  | 3.06  | 2.96  |
| 16   | 6.12                                     | 4.69  | 4.08  | 3.73  | 3.50  | 3.34  | 3.22  | 3.12  | 3.05  | 2.99  | 2.89  |
| 17   | 6.04                                     | 4.62  | 4.01  | 3.66  | 3.44  | 3.28  | 3.16  | 3.06  | 2.98  | 2.92  | 2.82  |
| 18   | 5.98                                     | 4.56  | 3.95  | 3.61  | 3.38  | 3.22  | 3.10  | 3.01  | 2.93  | 2.87  | 2.77  |
| 19   | 5.92                                     | 4.51  | 3.90  | 3.56  | 3.33  | 3.17  | 3.05  | 2.96  | 2.88  | 2.82  | 2.72  |
| 20   | 5.87                                     | 4.46  | 3.86  | 3.51  | 3.29  | 3.13  | 3.01  | 2.91  | 2.84  | 2.77  | 2.68  |
| 21   | 5.83                                     | 4.42  | 3.82  | 3.48  | 3.25  | 3.09  | 2.97  | 2.87  | 2.80  | 2.73  | 2.64  |
| 22   | 5.79                                     | 4.38  | 3.78  | 3.44  | 3.22  | 3.05  | 2.93  | 2.84  | 2.76  | 2.70  | 2.60  |
| 23   | 5.75                                     | 4.35  | 3.75  | 3.41  | 3.18  | 3.02  | 2.90  | 2.81  | 2.73  | 2.67  | 2.57  |
| 24   | 5.72                                     | 4.32  | 3.72  | 3.38  | 3.15  | 2.99  | 2.87  | 2.78  | 2.70  | 2.64  | 2.54  |
| 25   | 5.69                                     | 4.29  | 3.69  | 3.35  | 3.13  | 2.97  | 2.85  | 2.75  | 2.68  | 2.61  | 2.51  |
| 26   | 5.66                                     | 4.27  | 3.67  | 3.33  | 3.10  | 2.94  | 2.82  | 2.73  | 2.65  | 2.59  | 2.49  |
| 27   | 5.63                                     | 4.24  | 3.65  | 3.31  | 3.08  | 2.92  | 2.80  | 2.71  | 2.63  | 2.57  | 2.47  |
| 28   | 5.61                                     | 4.22  | 3.63  | 3.29  | 3.06  | 2.90  | 2.78  | 2.69  | 2.61  | 2.55  | 2.45  |
| 29   | 5.59                                     | 4.20  | 3.61  | 3.27  | 3.04  | 2.88  | 2.76  | 2.67  | 2.59  | 2.53  | 2.43  |
| 30   | 5.57                                     | 4.18  | 3.59  | 3.25  | 3.03  | 2.87  | 2.75  | 2.65  | 2.57  | 2.51  | 2.41  |
| 40   | 5.42                                     | 4.05  | 3.46  | 3.13  | 2.90  | 2.74  | 2.62  | 2.53  | 2.45  | 2.39  | 2.29  |
| 50   | 5.34                                     | 3.97  | 3.39  | 3.05  | 2.83  | 2.67  | 2.55  | 2.46  | 2.38  | 2.32  | 2.22  |
| 60   | 5.29                                     | 3.93  | 3.34  | 3.01  | 2.79  | 2.63  | 2.51  | 2.41  | 2.33  | 2.27  | 2.17  |
| ∞  | 5.15                                     | 3.80  | 3.23  | 2.89  | 2.67  | 2.52  | 2.39  | 2.30  | 2.22  | 2.16  | 2.05  |
| ∞  | 5.10                                     | 3.76  | 3.18  | 2.85  | 2.63  | 2.47  | 2.35  | 2.26  | 2.18  | 2.11  | 2.01  |
| ∞  | 2.71                                     | 5.02  | 3.69  | 3.12  | 2.79  | 2.57  | 2.41  | 2.29  | 2.19  | 2.11  | 2.05  |

| Denominator<br>Degrees of<br>Freedom ( $v_2$ ) | Numerator Degrees of Freedom ( $v_1$ ) |       |       |       |       |       |       |       |       |       |          |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|  | 15                                     | 20    | 25    | 30    | 35    | 40    | 50    | 60    | 120   | 200   | $\infty$ |
| 1  | 984.9                                  | 993.1 | 998.1 | 1001  | 1004  | 1006  | 1008  | 1010  | 1014  | 1016  | 1018     |
| 2  | 39.43                                  | 39.45 | 39.46 | 39.46 | 39.47 | 39.47 | 39.48 | 39.48 | 39.49 | 39.49 | 39.50    |
| 3  | 14.25                                  | 14.17 | 14.12 | 14.08 | 14.06 | 14.04 | 14.01 | 13.99 | 13.95 | 13.93 | 13.90    |
| 4  | 8.66                                   | 8.56  | 8.50  | 8.46  | 8.43  | 8.41  | 8.38  | 8.36  | 8.31  | 8.29  | 8.26     |
| 5  | 6.43                                   | 6.33  | 6.27  | 6.23  | 6.20  | 6.18  | 6.14  | 6.12  | 6.07  | 6.05  | 6.02     |
| 6  | 5.27                                   | 5.17  | 5.11  | 5.07  | 5.04  | 5.01  | 4.98  | 4.96  | 4.90  | 4.88  | 4.85     |
| 7  | 4.57                                   | 4.47  | 4.40  | 4.36  | 4.33  | 4.31  | 4.28  | 4.25  | 4.20  | 4.18  | 4.14     |
| 8  | 4.10                                   | 4.00  | 3.94  | 3.89  | 3.86  | 3.84  | 3.81  | 3.78  | 3.73  | 3.70  | 3.67     |
| 9  | 3.77                                   | 3.67  | 3.60  | 3.56  | 3.53  | 3.51  | 3.47  | 3.45  | 3.39  | 3.37  | 3.33     |
| 10   | 3.52                                   | 3.42  | 3.35  | 3.31  | 3.28  | 3.26  | 3.22  | 3.20  | 3.14  | 3.12  | 3.08     |
| 11   | 3.33                                   | 3.23  | 3.16  | 3.12  | 3.09  | 3.06  | 3.03  | 3.00  | 2.94  | 2.92  | 2.88     |
| 12   | 3.18                                   | 3.07  | 3.01  | 2.96  | 2.93  | 2.91  | 2.87  | 2.85  | 2.79  | 2.76  | 2.73     |
| 13   | 3.05                                   | 2.95  | 2.88  | 2.84  | 2.80  | 2.78  | 2.74  | 2.72  | 2.66  | 2.63  | 2.60     |
| 14   | 2.95                                   | 2.84  | 2.78  | 2.73  | 2.70  | 2.67  | 2.64  | 2.61  | 2.55  | 2.53  | 2.49     |
| 15   | 2.86                                   | 2.76  | 2.69  | 2.64  | 2.61  | 2.59  | 2.55  | 2.52  | 2.46  | 2.44  | 2.40     |
| 16   | 2.79                                   | 2.68  | 2.61  | 2.57  | 2.53  | 2.51  | 2.47  | 2.45  | 2.38  | 2.36  | 2.32     |
| 17   | 2.72                                   | 2.62  | 2.55  | 2.50  | 2.47  | 2.44  | 2.41  | 2.38  | 2.32  | 2.29  | 2.25     |
| 18   | 2.67                                   | 2.56  | 2.49  | 2.44  | 2.41  | 2.38  | 2.35  | 2.32  | 2.26  | 2.23  | 2.19     |
| 19   | 2.62                                   | 2.51  | 2.44  | 2.39  | 2.36  | 2.33  | 2.30  | 2.27  | 2.20  | 2.18  | 2.13     |
| 20   | 2.57                                   | 2.46  | 2.40  | 2.35  | 2.31  | 2.29  | 2.25  | 2.22  | 2.16  | 2.13  | 2.09     |
| 21   | 2.53                                   | 2.42  | 2.36  | 2.31  | 2.27  | 2.25  | 2.21  | 2.18  | 2.11  | 2.09  | 2.04     |
| 22   | 2.50                                   | 2.39  | 2.32  | 2.27  | 2.24  | 2.21  | 2.17  | 2.14  | 2.08  | 2.05  | 2.01     |
| 23   | 2.47                                   | 2.36  | 2.29  | 2.24  | 2.20  | 2.18  | 2.14  | 2.11  | 2.04  | 2.01  | 1.97     |
| 24   | 2.44                                   | 2.33  | 2.26  | 2.21  | 2.17  | 2.15  | 2.11  | 2.08  | 2.01  | 1.98  | 1.94     |
| 25   | 2.41                                   | 2.30  | 2.23  | 2.18  | 2.15  | 2.12  | 2.08  | 2.05  | 1.98  | 1.95  | 1.91     |
| 26   | 2.39                                   | 2.28  | 2.21  | 2.16  | 2.12  | 2.09  | 2.05  | 2.03  | 1.95  | 1.92  | 1.88     |
| 27   | 2.36                                   | 2.25  | 2.18  | 2.13  | 2.10  | 2.07  | 2.03  | 2.00  | 1.93  | 1.90  | 1.85     |
| 28   | 2.34                                   | 2.23  | 2.16  | 2.11  | 2.08  | 2.05  | 2.01  | 1.98  | 1.91  | 1.88  | 1.83     |
| 29   | 2.32                                   | 2.21  | 2.14  | 2.09  | 2.06  | 2.03  | 1.99  | 1.96  | 1.89  | 1.86  | 1.81     |
| 30   | 2.31                                   | 2.20  | 2.12  | 2.07  | 2.04  | 2.01  | 1.97  | 1.94  | 1.87  | 1.84  | 1.79     |
| 40   | 2.18                                   | 2.07  | 1.99  | 1.94  | 1.90  | 1.88  | 1.83  | 1.80  | 1.72  | 1.69  | 1.64     |
| 50   | 2.11                                   | 1.99  | 1.92  | 1.87  | 1.83  | 1.80  | 1.75  | 1.72  | 1.64  | 1.60  | 1.55     |
| 60   | 2.06                                   | 1.94  | 1.87  | 1.82  | 1.78  | 1.74  | 1.70  | 1.67  | 1.58  | 1.54  | 1.48     |
| 120  | 1.94                                   | 1.82  | 1.75  | 1.69  | 1.65  | 1.61  | 1.56  | 1.53  | 1.43  | 1.39  | 1.31     |
| 200  | 1.90                                   | 1.78  | 1.70  | 1.64  | 1.60  | 1.56  | 1.51  | 1.47  | 1.37  | 1.32  | 1.23     |
| $\infty$                                       | 2.71                                   | 1.83  | 1.71  | 1.63  | 1.57  | 1.52  | 1.48  | 1.43  | 1.39  | 1.27  | 1.21     |

# CRITICAL VALUES OF $F_{.01}$

This table shows the 1 percent right-tail critical values of  $F$  for the stated degrees of freedom ( $\nu$ ).



| Denominator<br>Degrees of<br>Freedom ( $\nu_2$ ) | Numerator Degrees of Freedom ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |       |  |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|
|  | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    |  |
| 1  | 4052                                     | 4999  | 5404  | 5624  | 5764  | 5859  | 5928  | 5981  | 6022  | 6056  | 6107  |  |
| 2  | 98.50                                    | 99.00 | 99.16 | 99.25 | 99.30 | 99.33 | 99.36 | 99.38 | 99.39 | 99.40 | 99.42 |  |
| 3  | 34.12                                    | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.34 | 27.23 | 27.05 |  |
| 4  | 21.20                                    | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 |  |
| 5  | 16.26                                    | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89  |  |
| 6  | 13.75                                    | 10.92 | 9.78  | 9.15  | 8.75  | 8.47  | 8.26  | 8.10  | 7.98  | 7.87  | 7.72  |  |
| 7  | 12.25                                    | 9.55  | 8.45  | 7.85  | 7.46  | 7.19  | 6.99  | 6.84  | 6.72  | 6.62  | 6.47  |  |
| 8  | 11.26                                    | 8.65  | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  | 5.81  | 5.67  |  |
| 9  | 10.56                                    | 8.02  | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  | 5.26  | 5.11  |  |
| 10   | 10.04                                    | 7.56  | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  | 4.85  | 4.71  |  |
| 11   | 9.65                                     | 7.21  | 6.22  | 5.67  | 5.32  | 5.07  | 4.89  | 4.74  | 4.63  | 4.54  | 4.40  |  |
| 12   | 9.33                                     | 6.93  | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  | 4.30  | 4.16  |  |
| 13   | 9.07                                     | 6.70  | 5.74  | 5.21  | 4.86  | 4.62  | 4.44  | 4.30  | 4.19  | 4.10  | 3.96  |  |
| 14   | 8.86                                     | 6.51  | 5.56  | 5.04  | 4.69  | 4.46  | 4.28  | 4.14  | 4.03  | 3.94  | 3.80  |  |
| 15   | 8.68                                     | 6.36  | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  | 3.80  | 3.67  |  |
| 16   | 8.53                                     | 6.23  | 5.29  | 4.77  | 4.44  | 4.20  | 4.03  | 3.89  | 3.78  | 3.69  | 3.55  |  |
| 17   | 8.40                                     | 6.11  | 5.19  | 4.67  | 4.34  | 4.10  | 3.93  | 3.79  | 3.68  | 3.59  | 3.46  |  |
| 18   | 8.29                                     | 6.01  | 5.09  | 4.58  | 4.25  | 4.01  | 3.84  | 3.71  | 3.60  | 3.51  | 3.37  |  |
| 19   | 8.18                                     | 5.93  | 5.01  | 4.50  | 4.17  | 3.94  | 3.77  | 3.63  | 3.52  | 3.43  | 3.30  |  |
| 20   | 8.10                                     | 5.85  | 4.94  | 4.43  | 4.10  | 3.87  | 3.70  | 3.56  | 3.46  | 3.37  | 3.23  |  |
| 21   | 8.02                                     | 5.78  | 4.87  | 4.37  | 4.04  | 3.81  | 3.64  | 3.51  | 3.40  | 3.31  | 3.17  |  |
| 22   | 7.95                                     | 5.72  | 4.82  | 4.31  | 3.99  | 3.76  | 3.59  | 3.45  | 3.35  | 3.26  | 3.12  |  |
| 23   | 7.88                                     | 5.66  | 4.76  | 4.26  | 3.94  | 3.71  | 3.54  | 3.41  | 3.30  | 3.21  | 3.07  |  |
| 24   | 7.82                                     | 5.61  | 4.72  | 4.22  | 3.90  | 3.67  | 3.50  | 3.36  | 3.26  | 3.17  | 3.03  |  |
| 25   | 7.77                                     | 5.57  | 4.68  | 4.18  | 3.85  | 3.63  | 3.46  | 3.32  | 3.22  | 3.13  | 2.99  |  |
| 26   | 7.72                                     | 5.53  | 4.64  | 4.14  | 3.82  | 3.59  | 3.42  | 3.29  | 3.18  | 3.09  | 2.96  |  |
| 27   | 7.68                                     | 5.49  | 4.60  | 4.11  | 3.78  | 3.56  | 3.39  | 3.26  | 3.15  | 3.06  | 2.93  |  |
| 28   | 7.64                                     | 5.45  | 4.57  | 4.07  | 3.75  | 3.53  | 3.36  | 3.23  | 3.12  | 3.03  | 2.90  |  |
| 29   | 7.60                                     | 5.42  | 4.54  | 4.04  | 3.73  | 3.50  | 3.33  | 3.20  | 3.09  | 3.00  | 2.87  |  |
| 30   | 7.56                                     | 5.39  | 4.51  | 4.02  | 3.70  | 3.47  | 3.30  | 3.17  | 3.07  | 2.98  | 2.84  |  |
| 40   | 7.31                                     | 5.18  | 4.31  | 3.83  | 3.51  | 3.29  | 3.12  | 2.99  | 2.89  | 2.80  | 2.66  |  |
| 50   | 7.17                                     | 5.06  | 4.20  | 3.72  | 3.41  | 3.19  | 3.02  | 2.89  | 2.78  | 2.70  | 2.56  |  |
| 60   | 7.08                                     | 4.98  | 4.13  | 3.65  | 3.34  | 3.12  | 2.95  | 2.82  | 2.72  | 2.63  | 2.50  |  |
| 120  | 6.85                                     | 4.79  | 3.95  | 3.48  | 3.17  | 2.96  | 2.79  | 2.66  | 2.56  | 2.47  | 2.34  |  |
| 200  | 6.76                                     | 4.71  | 3.88  | 3.41  | 3.11  | 2.89  | 2.73  | 2.60  | 2.50  | 2.41  | 2.27  |  |
| $\infty$   | 2.71                                     | 6.63  | 4.61  | 3.78  | 3.32  | 3.02  | 2.80  | 2.64  | 2.51  | 2.41  | 2.32  |  |

| Denominator<br>Degrees of<br>Freedom ( $v_2$ ) | Numerator Degrees of Freedom ( $v_1$ ) |       |       |       |       |       |       |       |       |       |          |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|  | 15                                     | 20    | 25    | 30    | 35    | 40    | 50    | 60    | 120   | 200   | $\infty$ |
| 1  | 6157                                   | 6209  | 6240  | 6260  | 6275  | 6286  | 6302  | 6313  | 6340  | 6350  | 6366     |
| 2  | 99.43                                  | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.48 | 99.48 | 99.49 | 99.49 | 99.50    |
| 3  | 26.87                                  | 26.69 | 26.58 | 26.50 | 26.45 | 26.41 | 26.35 | 26.32 | 26.22 | 26.18 | 26.13    |
| 4  | 14.20                                  | 14.02 | 13.91 | 13.84 | 13.79 | 13.75 | 13.69 | 13.65 | 13.56 | 13.52 | 13.47    |
| 5  | 9.72                                   | 9.55  | 9.45  | 9.38  | 9.33  | 9.29  | 9.24  | 9.20  | 9.11  | 9.08  | 9.02     |
| 6  | 7.56                                   | 7.40  | 7.30  | 7.23  | 7.18  | 7.14  | 7.09  | 7.06  | 6.97  | 6.93  | 6.88     |
| 7  | 6.31                                   | 6.16  | 6.06  | 5.99  | 5.94  | 5.91  | 5.86  | 5.82  | 5.74  | 5.70  | 5.65     |
| 8  | 5.52                                   | 5.36  | 5.26  | 5.20  | 5.15  | 5.12  | 5.07  | 5.03  | 4.95  | 4.91  | 4.86     |
| 9  | 4.96                                   | 4.81  | 4.71  | 4.65  | 4.60  | 4.57  | 4.52  | 4.48  | 4.40  | 4.36  | 4.31     |
| 10   | 4.56                                   | 4.41  | 4.31  | 4.25  | 4.20  | 4.17  | 4.12  | 4.08  | 4.00  | 3.96  | 3.91     |
| 11   | 4.25                                   | 4.10  | 4.01  | 3.94  | 3.89  | 3.86  | 3.81  | 3.78  | 3.69  | 3.66  | 3.60     |
| 12   | 4.01                                   | 3.86  | 3.76  | 3.70  | 3.65  | 3.62  | 3.57  | 3.54  | 3.45  | 3.41  | 3.36     |
| 13   | 3.82                                   | 3.66  | 3.57  | 3.51  | 3.46  | 3.43  | 3.38  | 3.34  | 3.25  | 3.22  | 3.17     |
| 14   | 3.66                                   | 3.51  | 3.41  | 3.35  | 3.30  | 3.27  | 3.22  | 3.18  | 3.09  | 3.06  | 3.01     |
| 15   | 3.52                                   | 3.37  | 3.28  | 3.21  | 3.17  | 3.13  | 3.08  | 3.05  | 2.96  | 2.92  | 2.87     |
| 16   | 3.41                                   | 3.26  | 3.16  | 3.10  | 3.05  | 3.02  | 2.97  | 2.93  | 2.84  | 2.81  | 2.76     |
| 17   | 3.31                                   | 3.16  | 3.07  | 3.00  | 2.96  | 2.92  | 2.87  | 2.83  | 2.75  | 2.71  | 2.66     |
| 18   | 3.23                                   | 3.08  | 2.98  | 2.92  | 2.87  | 2.84  | 2.78  | 2.75  | 2.66  | 2.62  | 2.57     |
| 19   | 3.15                                   | 3.00  | 2.91  | 2.84  | 2.80  | 2.76  | 2.71  | 2.67  | 2.58  | 2.55  | 2.49     |
| 20   | 3.09                                   | 2.94  | 2.84  | 2.78  | 2.73  | 2.69  | 2.64  | 2.61  | 2.52  | 2.48  | 2.42     |
| 21   | 3.03                                   | 2.88  | 2.79  | 2.72  | 2.67  | 2.64  | 2.58  | 2.55  | 2.46  | 2.42  | 2.36     |
| 22   | 2.98                                   | 2.83  | 2.73  | 2.67  | 2.62  | 2.58  | 2.53  | 2.50  | 2.40  | 2.36  | 2.31     |
| 23   | 2.93                                   | 2.78  | 2.69  | 2.62  | 2.57  | 2.54  | 2.48  | 2.45  | 2.35  | 2.32  | 2.26     |
| 24   | 2.89                                   | 2.74  | 2.64  | 2.58  | 2.53  | 2.49  | 2.44  | 2.40  | 2.31  | 2.27  | 2.21     |
| 25   | 2.85                                   | 2.70  | 2.60  | 2.54  | 2.49  | 2.45  | 2.40  | 2.36  | 2.27  | 2.23  | 2.17     |
| 26   | 2.81                                   | 2.66  | 2.57  | 2.50  | 2.45  | 2.42  | 2.36  | 2.33  | 2.23  | 2.19  | 2.13     |
| 27   | 2.78                                   | 2.63  | 2.54  | 2.47  | 2.42  | 2.38  | 2.33  | 2.29  | 2.20  | 2.16  | 2.10     |
| 28   | 2.75                                   | 2.60  | 2.51  | 2.44  | 2.39  | 2.35  | 2.30  | 2.26  | 2.17  | 2.13  | 2.07     |
| 29   | 2.73                                   | 2.57  | 2.48  | 2.41  | 2.36  | 2.33  | 2.27  | 2.23  | 2.14  | 2.10  | 2.04     |
| 30   | 2.70                                   | 2.55  | 2.45  | 2.39  | 2.34  | 2.30  | 2.25  | 2.21  | 2.11  | 2.07  | 2.01     |
| 40   | 2.52                                   | 2.37  | 2.27  | 2.20  | 2.15  | 2.11  | 2.06  | 2.02  | 1.92  | 1.87  | 1.81     |
| 50   | 2.42                                   | 2.27  | 2.17  | 2.10  | 2.05  | 2.01  | 1.95  | 1.91  | 1.80  | 1.76  | 1.69     |
| 60   | 2.35                                   | 2.20  | 2.10  | 2.03  | 1.98  | 1.94  | 1.88  | 1.84  | 1.73  | 1.68  | 1.60     |
| 120  | 2.19                                   | 2.03  | 1.93  | 1.86  | 1.81  | 1.76  | 1.70  | 1.66  | 1.53  | 1.48  | 1.38     |
| 200  | 2.13                                   | 1.97  | 1.87  | 1.79  | 1.74  | 1.69  | 1.63  | 1.58  | 1.45  | 1.39  | 1.28     |
| $\infty$                                       | 2.71                                   | 2.04  | 1.88  | 1.77  | 1.70  | 1.64  | 1.59  | 1.52  | 1.47  | 1.32  | 1.25     |