Research Article

Production-Inventory Model For Noninstantaneous Deteriorating Inventory Items With Stock Dependent, Price Decreasing Demand, and Fully Backlogged Under Inflation

Shiv Kumar*

Abstract

In this article, an inventory model for noninstantaneous deteriorating items with stock-dependent, price-decreasing demand in an inflationary environment is considered. Deterioration rate is studied with the help of Heaviside's function. In this inventory model, shortages are permitted with a full backlog. Techniques are used to minimize the total cost in this research. Numerical illustrations under different cases and sensitivity analysis of the proposed model are presented.

Keywords:

Inventory, Production, Deterioration, Stock, Demand.

Introduction

In traditional inventory system, many researchers analyzed the inventory system without assuming the important role of inflation. In the business, inflation plays an important role to carry on for a long time. Therefore, the effect of inflation on development of the inventory system can not be neglected. The ups and downs in the market are affected by different situations, but it is most affected by inflation. Inflation is a global event in the current times. Due to inflation, normally the price of the items and the service of an economics over a period of time are raised. When service and products are in high demand due to inflation, then it effects the economy, social, moral, and political situation of society. When an inventory system is scrutinized, then it is observed that price and stock level of

* *Faculty, Indian Institute of Technology (ISM) Dhanbad, Jharkhand, India*

inventory is an important factor for any product to carry on the business. Besides, due to the affect of deterioration, it refers to decay, spoilage or damage. In case of products like foods, drugs, films, electronics components, chemical and radio-active substances, deterioration may occur during the general period of storage. Basu et al. (2007) derived an inflationary inventory model with time dependent demand with Weibull distribution deterioration and partial backlogging under permissible delay in payments. Buzacott (1975) explained an inventory system with different pricing policies under inflation. Pioneer work is preceded by Buzacott.

Choudri et al. (2016) explained production- inventory model for deteriorating items with constant demand under the effect of inflation and time-value of money.

To evaluate the delivery lot size items have better solution ideas and minimize the total cost of the entire supply chain, have been worked the number of deliveries per production batch cycle by Chang (2014). Chang et al. (2006) presented an EOQ model for perishable products under constant deterioration with the stock-dependent selling rate, time-dependent partial backlogging, and unfulfilled demand in case of backlogging. Dye (2005) explained instantaneous stock-dependent demand and time-proportional backlogging rate. This is a realistic theory applicable in practice. Datta & Pal (1991)derived a finite time horizon inventory system under the effects of inflation and time value of money in which they considered linear time-dependent demand rate and shortages. They also observed two solution procedures with and without shortages. There is a restriction of the equal replenishment cycle in this. Hishamuddin et al. (2012) examined different types of disruption recovery models for a single stage production and when production is increasing period, then the production is disrupted for a given period of time. Hou (2006) explained an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation as well as time discounting. Khurana et al. (2015) explained two types of cost calculated in an inventory model—the first is the vendor cost and the second is the retailer cost under the effect of inflation environment. Kumar & Singh (2015) derived an optimal replenishment policy for noninstantaneous deteriorating items with stock dependent, price decreasing demand, and partial backlogging. Khedlerkar et al. (2012) considered variable demand and the uniform production rate. Manna & Chaudhuri (2006) investigated ramp type demand rate time dependent deterioration rate, unit production cost and shortages.. Raj (2013) worked on an integrated inventory optimization for deteriorating product under multi-echelon just in time. Manna & Chiang (2010) presented two deterministic economic production quantity models for Weibull distribution deteriorating items with ramp type demand rate. Panda et al. (2008) derived a single period inventory model with imperfect production and stochastic demand under chance and imprecise constraints. A multi-echelon supply chain model for rework able items in multiple-markets with supply disruption was scrutinized by Pal et al. (2012). Ray & Chaudhauri (1997) investigated the effect of time value of money and inflation in their inventory model. Rivera- Gomez et al. (2013) worked on a model in which production as well as quality control of different types of good-well in manufacturing organization/industry. Sana et al(2015) explained an inventory system of pharmaceutical products where the demand rate of the consumers increases with the volume of the initiatives of the sales team and there is deterioration of the various items depending on on-hand inventory. Sivashankari (2015) describesed production inventory model with deteriorating items and price. Sivashankari is assumed constant demand. Singh et al. (2013) introduced an EOQ model with volume agility, variable demand rate, Weibull deterioration rate, and inflation. Tayal et al. (2015) introduced preservation technology for production inventory system of perishable products with trade credit period. In this inventory system, by using a preservation technology, existing rate of deterioration was reduced. Young & Ju (2010) developed a production model for deteriorating inventory items with production disruption. Yang et al. (2010) presented an economic order quantity model under inflation with shortages and partial backlogging.

Research Gap and Contribution of Study

In the proposed model, dependent demand with decreasing selling price under the inflation as well as noninstantaneous deterioration was considered. The goal is to minimize the total average cost of the product where lack of product is permitted for the optimal policies and the corresponding cost in the proposed model.

Proposed Model

The variation of inventory level of a cycle is depicted in figure1. Initially at time, start the production and demand both of an items, where production is greater than demand. The total amount of inventory level in interval is represented by. At point A, the production is stopped of items. In interval, inventory level is represented by. In the interval, demand and deterioration are occurred. At time, when inventory level reaches zero. In the interval, only backlogged demand is started, where inventory level is present by. In the interval, production and backlogged demand of items are occurred and inventory level is represented by.

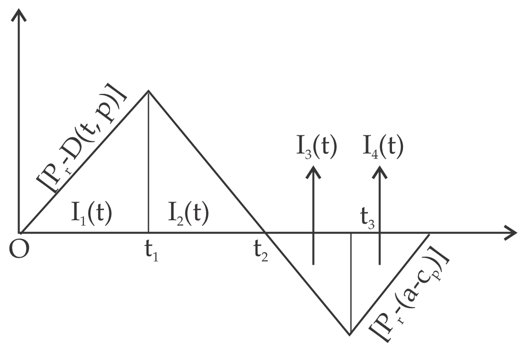

Figure I. Graph of Inventory System

Assumptions

The following expressions Buildup the construction are proposed inventory model

(1) The single product and a single manufacturing facility produce the demand for a finite time horizon.

(2) Normal production rate is greater than the demand rate.
(3) Demand rate is stock dependent and a price decreasing demand.
(4) Deterioration rate is noninstantaneous.
(5) The planning horizon is finite.
(6) Lead time is zero
(7) Replenishment rate is infinite.
(8) Shortages are allowed.

Notations

The basic parameters are as follows:

a - Initial demand of items

b - Parameter of demand governing increasing b>0 or b<0 trend, b<a

P - Deterministic normal production rate and P,-D(t, p)>0.

D(t, p) Demand rate of products is the function of stock dependent and price decreasing dependent and this is defined as follows:

$$D(t,p)=\begin{cases} a+bI(t)-cp, & I(t)\geq 0 \\ a-cp, & I(t)\leq 0 \end{cases}$$

Where a, b, c are positive constant and p is selling price per unit item and I(t) is the inventory level at any time t.

$\theta(t)$ The deterioration rate of the items is assumed to be governed by the

$$\theta(t)=\theta H(t-t_1)=\begin{cases} \theta & t>t_1 \\ 0 & t<t_1 \end{cases}$$

Where t is the time measured from the instant of arrival of replenishment $\theta\ (0<\theta<1)$ is constant and is $H(t-t_1)$ Heaviside's function.

TAC The total average cost per unit time of the inventory system

$I_1(t)$ Inventory level at any time t, $0<t<t_1$

$I_2(t)$ Inventory level at any time t, $t_1<t<t_2$

$I_3(t)$ Inventory level at any time t, $t_2\leq t\leq t_3$

$I_4(t)$ Inventory level at any time t, $t_3\leq t\leq T$

r Rate of inflation

t_1 The length of time in which the product exhibits no deterioration
t_2 Time when inventory level reaches to zero
t_3 Time when inventory level reaches to fully backlogged
T The length of the replenishment cycle time
C_S Shortage cost per unit per unit time of the item
C_H Holding cost per unit per unit time of the item
C_D Deterioration cost per unit of the item
C_P Purchase cost per unit of the item
p Selling price per unit of the item
A Ordering cost per order of the item
t^*_1 The optimal length of time in which the product exhibits no deterioration
t^*_2 The optimal time when inventory level reaches zero
t^*_2 The optimal time when inventory level reaches fully backlogged
T^* The optimal the length of the replenishment cycle

Mathematical Formulations and Analysis

When the production rate and demand rate in the interval, then there is no deterioration of the items and the dynamic of inventory level at any time, is governed by the following differential equations:

$$\frac{dI_1(t)}{dt} = P_r - \left(a + bI_1(t) - cp\right), \qquad 0 \le t \le t_1 \tag{1}$$

With initial condition $I_1(0) = 0$ when $t = 0$;

At time $t = t_1$ production is stopped and there is only demand and deterioration occurred in the inventory system, hence the differential equation can be written as follows

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -\left(a + bI_2(t) - cp\right), \qquad t_1 \le t \le t_2 \tag{2}$$

With boundary condition $I_2(t) = 0$, when $t = t_2$;
The solutions of differential equations (1) and (2) are given by:

$$I_1(t) = \frac{(P_r - a + cp)\left(1 - e^{-bt}\right)}{b}, \quad 0 \le t \le t_1 \tag{3}$$

$$I_2(t) = \frac{(-a + cp) + (1 - e^{(\theta+b)(t_2-t)})}{(\theta + b)}, \quad t_1 \le t \le t_2 \tag{4}$$

From Figure 1, it can be considered that the continuity of inventory level such that, $I_1(t) = I_2(t)$ when $t = t_1$ implies that:

$$t_2 = t_1 + \frac{1}{(\theta + b)} \log\left[-1 + \frac{(\theta + b)\left(1 - e^{-bt_1}\right)}{(-ab + bcp)} \right] \qquad (5)$$

In interval, $[t_2, t_3]$ the variation of shortage level is taken as time during the inventory level and it is fully backlogged, then it can be described by the following differential equation

$$\frac{dI_3(t)}{dt} = -(a - cp), \qquad t_2 \le t \le t_3 \qquad (6)$$

With boundary condition $I_3(t) = 0$, when $t = t_2$;
The solutions of differential equations (6) are given by:

$$I_3(t) = (-a + cp)(t - t_2) \qquad (7)$$

With boundary condition $I_4(t) = 0$ when $t = T$ Since we observed that at $t = t_3$ production start again and backlogging is cleared at $t = T$, then it can be described by the following differential equation:

$$I_4(t) = (P_r - a + cp)(t - T) \qquad (8)$$

At $t = t_3$, $I_3(t) = I_4(t)$ from equations (7) and (8) the following is derived:

$$t_3 = \frac{T(P_r - a + cp) - t_2(-a + cp)}{P_r} \qquad (9)$$

The total relevant inventory cost per cycle is as follows:

(i) is the setup cost per cycle.

(ii) Inventory holding/storage cost of the system is given by the following:

$$HC = c_h\left[\int_0^{t_1} I_1(t)e^{-rt}dt + \int_{t_1}^{t_2} I_2(t)e^{-rt}dt\right] = c_h\left[\left(\frac{P_r - a + cp}{b}\right)\left\{\frac{b\left(1 - e^{-rt_1}\right) + re^{-rt_1}\left(e^{-bt_1} - 1\right)}{(b + r)r}\right\} + \left(\frac{-a + cp}{(b + r)r}\right)\left(e^{-rt_1} - e^{-rt_2}\right) - \left(\left(\frac{-a + cp}{(b + \theta)(b + \theta + r)}\right)\left(e^{(b+\theta)(t_2 - t_1) - rt_2} - e^{-rt_1}\right)\right)\right]$$

(iii) The deterioration cost per cycle is represented by the following:

$$DC = c_d\left[\int_0^{t_1}\theta(t)I_1(t)e^{-rt}dt + \int_{t_1}^{t_2}\theta(t)I_2(t)e^{-rt}dt\right] = \theta c_d\left[\left(\frac{-a + cp}{(b + \theta)r}\right)\left(e^{-rt_1} - e^{-rt_2}\right) - \left(\frac{-a + cp}{(b + \theta)(b + \theta + r)}\right)\left(e^{(b+\theta)(t_2 - t_1) - rt_2} - e^{-rt_1}\right)\right]$$

(iv) The shortage cost in the entire cycle is described by the following:

$$SC = -c_s\int_{t_1}^{T} I_3(t)e^{-rt}dt = -c_s\left[\left(\frac{-a + cp}{r^2}\right)\left\{e^{-rt_3}\left(-1 + r(t_2 - t_3)\right) + e^{-rt_2}\right\} + \left(\frac{P_r - a + cp}{r^2}\right)\left\{e^{-rT}\left(-1 + r(t_3 - T)\right) + e^{-rt_3}\right\}\right]$$

Total profit of the system per unit time per cycle is given as follows:

$$TAC(t_1, t_2, t_3, T, b) = \frac{1}{T}[A + HC + DC + SC] \qquad (10)$$

Optimality of the Cost Function

There are calculated different optimal cost function of product under some special cases in proposed model.

Case 1

The cost of the system per unit time per cycle $TAC(t_1,t_2,t_3,T,b)$ is a function of only t_1.
In this case, the cost of the system per unit time per cycle is expressed as follows:

$$TAC(t_1)=\frac{1}{T}[A+HC+DC+SC] \quad (11)$$

The optimal value of t_1^* is obtained by satisfying the following necessary condition:

$\frac{\partial TAC(t_1)}{\partial t_1}=0$, along with the following sufficient condition $\frac{\partial^2 TAC}{\partial t_1^2}<0$

Case 2

The cost of the system per unit time per cycle $TAC(t_1,t_2,t_3,T,b)$ is function of only ..
In this case, the cost of the system per unit time per cycle is expressed as follows:

$$TAC(t_2)=\frac{1}{T}[A+HC+DC+SC] \quad (12)$$

The optimal value of t_2^* is obtained by satisfying the following necessary condition:

$\frac{\partial TAC(t_2)}{\partial t_2}=0$, along with the following sufficient condition $\frac{\partial^2 TAC}{\partial t_2^2}<0$

Case 3

The cost of the system per unit time per cycle $TAC(t_1,t_2,t_3,T,b)$ is a function of only t_3.
In this case, the cost of the system per unit time per cycle is expressed as follows:

$$TAC(t_3)=\frac{1}{T}[A+HC+DC+SC] \quad (13)$$

The optimal value of t_3^* is obtained by satisfying the following necessary condition:

$\frac{\partial TAC(t_3)}{\partial t_3}=0$, along with the following sufficient condition $\frac{\partial^2 TAC}{\partial t_3^2}<0$

Case 4

The cost of the system per unit time per cycle $TAC(t_1,t_2,t_3,T,b)$ is a function of only T.
In this case, the cost of the system per unit time per cycle is expressed as follows:

$$TAC(T)=\frac{1}{T}[A+HC+DC+SC] \quad (14)$$

The optimal value of T^* is obtained by satisfying the following necessary condition:

$\frac{\partial TAC(T)}{\partial T}=0$, along with the following sufficient condition $\frac{\partial^2 TAC}{\partial T^2}<0$

Case 5

The cost of the system per unit time per cycle $TAC(t_1,t_2,t_3,T,b)$ is a function of only .
In this case, the cost of the system per unit time per cycle is expressed as follows:

$$TAC(b)=\frac{1}{T}[A+HC+DC+SC] \quad (15)$$

The optimal value of b^* is obtained by satisfying the following necessary condition:

$\frac{\partial TAC(b)}{\partial b}=0$, along with the following sufficient condition $\frac{\partial^2 TAC}{\partial b}<0$

Numerical There are taken different numerical examples for propose inventory model.

Example 1 Numerical Illustration

For the numerical illustration, an inventory system with the following parameter in a proper unit was considered:
Where for t_2,t_3,T,b take fixed value.

$c_d=\$2/unit, c_h=\$4/per$ unit / $per\ unit\ time, c=100, p=\$13\ per$ unit / $per\ unit\ time,\ c_S=\$1/per\ unit/per\ unit\ time,$
$\theta=0.02, r=0.05, t_2=2$ month, $P_r=8800 t_3=3$ month, $T=4$ month, $a=100\ units/month, b=0.08, A=\$100/per\ order,$

Then using equation (11), and on solving a system of nonlinear equation by Newton Rapshon method we get the following:
Optimal time $t_1^*=\$0.4021$ month,
Optimal total average cost of the inventory system $TAC^*=\$\ 7693.99$,
Table 1 manifests the following facts:
1. Optimal total average cost TAC^* is highly sensitive when the change in the value of its parameters *a, b*, and *c* is done. It is slightly sensitive to the change in r and θ. It is moderately sensitive to changes in P_r.
2. t_1^* is highly sensitive to the change in parameter a,b and c whereas slightly sensitive to change in θ, r *and* P_r.

Example 2. Graphical Analysis

Using the derivation of Example 1 and taken the numerical values with the help of Mathematica software the graphical representation of the effect of time when on hand inventory production stops, on cost presented in Figure 2.

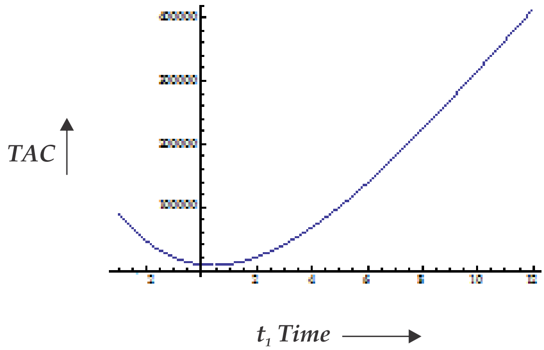

Figure 2. Variation of Profit with Respect to Price .

Example 3 Numerical Illustration

For the numerical illustration an inventory system with the following parameter in a proper unit was considered:
Where t_1, t_3, T, b take fixed value.
Then using equation (12) and on solving a system of nonlinear equation by Newton Rapshon method we get the following:

Optimal time

$$c_d = \$2/unit, c_h = \$3/\ per\ \text{unit}\ /\ per\ unit\ time, c = 50, p = \$13 per\ \text{unit}\ /\ per\ unit\ time, c_s = \$1/\ per\ unit/\ per\ unit\ time,$$
$$\theta = 0.02, r = 0.05, t_1 = 1\text{month}, P_r = 4500, t_3 = 2\ \text{month}, T = 3\text{month}, a = 600\ units/month,\ b = 0.48,\ A = \$100/\ per\ order,$$

Optimal total average cost of the inventory system $TAC^* = \$\ 1310.47$,
Table 2 manifests the following facts:

$$t_2^* = 1.73\ \text{month}$$

1. Optimal total average cost TAC^* is highly sensitive corresponding to the value of its parameters b and P_r.. It is slightly sensitive to the change in the parameter.
2. t_2^* is and highly sensitive to the change in parameter b whereas moderately sensitive to change in θ . It is also observed that for the value of parameter a, c and P_r there is no sensitive corresponding t_1^*.

Graphical Analysis

Using the derivation of Example 3 and taking the numerical values with the help of Mathematica software the graphical representation of the effect of time , on the total cost of inventory system is done in Figure 3.

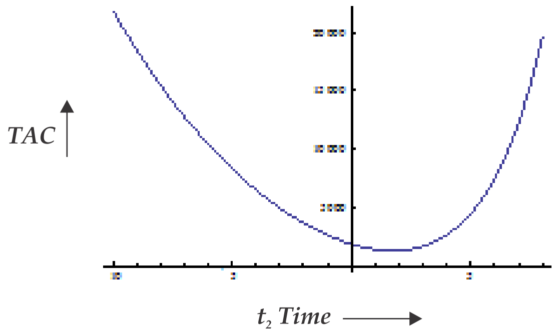

Figure 3. Variation of the Total Cost of Inventory System with Respect to Time t_2.

Example 4 Numerical Illustration

For the numerical illustration, an inventory system with the following parameter in a proper unit was considered:
Where t_1, t_2, T, b take fixed value.

$$c_d = \$1/unit, c_h = \$1/\ per\ \text{unit}\ /per\ unit\ time, c = 50, p = \$5\ per\ \text{unit}\ /\ per\ unit\ time,\ c_S = \$1/\ per\ unit/\ per\ unit\ time,$$
$$\theta = 0.02, r = 0.08,\ \ t_1 = 1\text{month},\ P_r = 4500, t_2 = 2\ \text{month},\ T = 3\ \text{month}, a = 600\ units/\ month,\ b = 0.48,\ A = \$400\ /\ per\ order,$$

Then using Equation (13) and on solving a system of nonlinear equation by Newton Rapshon method we get the following:

Optimal time $t_3^* = 2.12$ month
Optimal total average cost of the inventory system $TAC^* = \$\ 118.219$,

Graphical Analysis

Using derivation of Example 3 and taking the numerical values with the help of Mathematica software the graphical representation of the effect of time t_3, when on hand inventory reduce to zero, on profit is done in Figure 4.

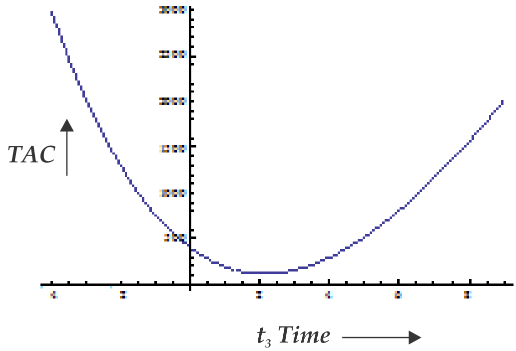

Figure 4. Variation of the Total Average Cost of the Inventory System with Respect to Time t_3.

Example 5 Numerical Illustration

For the numerical illustration, an inventory system with the following parameter in a proper unit is considered:
Where t_1, t_2, t_3, b take fixed value.

$c_d = \$4/\,unit, c_h = \$5/\,per$ unit $/\,per\ unit\ time, c = 50, p = \$10\,per$ unit $/\,per\ unit\ time,\ c_S = \$1/\,per\ unit/\,per\ unit\ time,$
$\theta = 0.02, r = 0.04,\ t_1 = 1$ month, $P_r = 1200, t_2 = 2$ month, $t_3 = 2.5$month, $a = 2500\ units/month,\ b = 0.48,\ A = \$50/\,per\ order,$

Then using equation (14), on solving system of a nonlinear equation by Newton Rapshon method by we get,
Optimal time $T^* = 2.5$ month,
Optimal total average cost of the inventory system $TAC^* = \$\ 207.545,$

Table 3 manifests the following facts:
1. Optimal average cost TAC^* is highly sensitive to the change in the value of its parameters b and P slightly sensitive to parameter c, r, P_r and θ.
2. T^* is not sensitive to the change in the parameter a, b, c, r, P_r and θ.

Using derivation of Example 4 and the numerical values taken with the help of Mathematica software the graphical representation of the effect of time T, when on hand inventory reduce to zero, on profit is done in Figure 5.

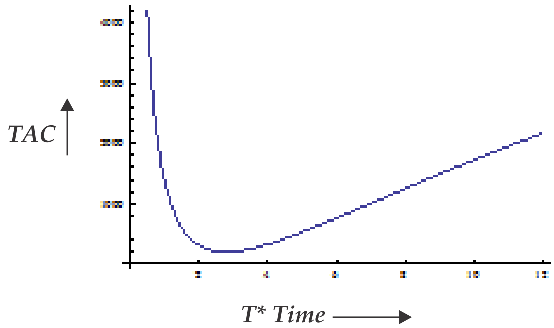

Figure 5. Variation of the Total Average Cost of the Inventory System with Respect to Time T.

Example 6
For the numerical illustration consider an inventory system with the following parameter in a proper unit:

$c_d = \$5 / unit, c_h = \$6 / per$ unit $/ per\ unit\ time, c = 40, p = \$8 per$ unit $/ per\ unit\ time,\ c_S = \$1 / per\ unit / per\ unit\ time,$
$\theta = 0.04, r = 0.01,\ t_1 = 1$ month, $P_r = 6000, t_2 = 2$ month, $t_3 = 4$month$, T = 5$month$, a = 2000\ units / month,\ A = \$5000 / per\ order,$

Where t_1, t_2, t_3, b take fixed value.
Then using Equation 15 and on solving system of a nonlinear equation by Newton Rapshon method by we get:
Optimal demand parameter $b^* = 0.8996,$
Optimal total average cost of the inventory system $TAC^* = \$\ 3683.25,$

Table 4 manifests the following facts:
1. Optimal total average cost TAC^* is highly sensitive to demand when the change in the value of its parameters a and b is done. It is slightly sensitive to the change in the parameter a, c and r. It is moderately sensitive to changes in θ.
2. b^* is highly sensitive to the change in parameter c and p whereas slightly sensitive to change in a and r as well as moderately sensitive to corresponding θ.

Graphical Analysis

Using derivation of Example 5 and the numerical values taken with the help of Mathematica software the graphical representation of the effect of time, when on hand inventory reduce to zero, on profit is done in Figure 6.

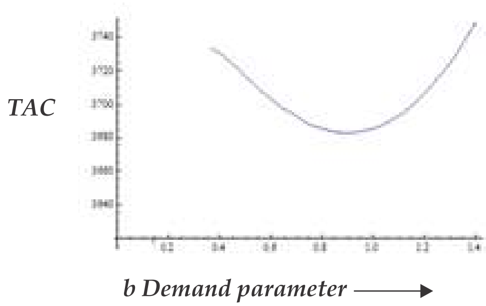

Figure 6. Variation of the Total Average Cost of the Inventory System with Respect to Demand Parameter *b* .

Conclusions

The optimal time to minimize the total cost per unit is determined and the proposed model is presented analytically and graphically. Sensitivity analysis shows that which parameters are responsible for effective changes in optimal cost and optimal parameters . This proposed model is very useful in the real market. It can be used for fashionable goods, clothes, vegetables and fruits, electronic component, and other products. Expected features of the proposed model are given by examples 1, 2, 3, 4, and 5. In tables 1, 2, 3 and 4, sensitivity analysis has also been discussed to study the effect of the pertinent parameters, viz., initial demand, the parameter of demand, the coefficient of selling price, the rate of inflation, deterioration rate, and production rate on optimal cost.

Limitations of the Study

In the present study of the inventory system there are some limitations, which are as follows:

- This inventory model reduces the business risk up to a great extent, but this study does not guarantee the elimination of business risk.
- This research is limited to the case when the production of goods in proposed model is allowed.
- In this study, the lead time is considered zero.
- This research is limited to the case when noninstantaneous deterioration rate is taken as Heaviside's function.

Future Scope

The proposed model can be further extended by including some extra realistic parts connected with the inventory model. Also stock dependent demand with time decreasing demand, quantity discount, Weibull deterioration, and so on may be changed in this proposed model.

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Table 1. Sensitivity Analysis by Changing the Specified Parameter by-50%, -

Parameters	% Changes	t_1^*	TAC^*
a	−50	+29.42	+30.13
	−25	+2.98	+3.11
	+25	−15.02	−15.87
	+50	−30.26	−32.31
b	−50	+113.39	+114.37
	−25	+43.75	+4152
	+25	−29.59	−32.61
	+50	−50.63	−53.16
c	−50	−80.55	−88.78
	−25	−39.52	−42.43
	+25	+38.02	+38.77
	+50	+74.83	+74.13

Table 2. Sensitivity Analysis by Changing the Specified Parameter by-50%, -

Parameters	% Changes	t_2^*	TAC^*
a	−50	0.00	+10.98
	−25	0.00	+5.49
	+25	0.00	−5.49
	+50	0.00	−10.98
b	−50	+76.88	−20.64
	−25	+20.90	+1.44
	+25	−10.11	−4.78
	+50	−15.83	−9.95
c	−50	0.00	−4.57
	−25	0.00	−2.89
	+25	0.00	+2.28
	+50	0.00	+4.57

θ	−50	+9.72	+21.41	θ	−50	+1.21	+2.54
	−25	+4.84	+10.05		−25	+0.59	+1.26
	+25	−4.60	−8.95		+25	−0.57	−1.26
	+50	−9.02	−16.97		+50	−1.16	−2.46
r	−50	−42.35	−49.62	r	−50	−14.36	−7.14
	−25	−15.89	−21.68		−25	−7.20	−3.32
	+25	+9.45	+17.50		+25	+7.27	+2.90
	+50	+14.79	+31.96		+50	+14.62	+2.40
r	−50	+91.34	−5.15	r	−50	0.00	−55.13
	−25	+31.33	−0.86		−25	0.00	−27.56
	+25	−19.27	−0.60		+25	0.00	+27.56
	+50	−32.30	−1.96		+50	0.00	+55.13

Table 3. Sensitivity Analysis by Changing the Specified Parameter by −50%, −25%, +25%, +50%

Table 4. Sensitivity Analysis by Changing the Specified Parameter by −50%, −25%, +25%, +50%

Parameters	% Changes	*	*	Parameters	% Changes	*	*
a	−50	0.00	+257.49	*a*	−50	+127.54	−30.86
	−25	0.00	+128.74		−25	+54.44	−13.38
	+25	0.00	−128.74		+25	−31.09	+17.09
	+50	0.00	−257.49		+50	−67.46	+15.43
b	−50	0.00	−1578.14	*c*	−50	−17.27	+3.64
	−25	0.00	−477.99		−25	−8.43	+1.85
	+25	0.00	+272.38		+25	+8.31	−1.93
	+50	0.00	+457.90		+50	+16.66	−3.94
c	−50	0.00	−51.49	θ	−50	+1.65	+0.02
	−25	0.00	−25.74		−25	+0.83	+0.01
	+25	0.00	+25.74		+25	−0.85	−0.01
	+50	0.00	+51.49		+50	−1.74	−0.03

θ	−50	0.00	−37.66	r	−50	+1.04	+0.13
	−25	0.00	−18.68		−25	+0.53	+0.06
	+25	0.00	+18.68		+25	−0.55	−0.06
	+50	0.00	+36.53		+50	−1.13	−0.13
r	−50	0.00	+376.76	r	−50	−93.16	−80.94
	−25	0.00	+182.32		−25	−61.14	−16.75
	+25	0.00	−170.97		+25	+35.57	+13.52
	+50	0.00	−331.31		+50	+61.78	+25.79
r	−50	0.00	−251.17				
	−25	0.00	−74.41				
	+25	0.00	+125.58				
	+50	0.00	+251.17				